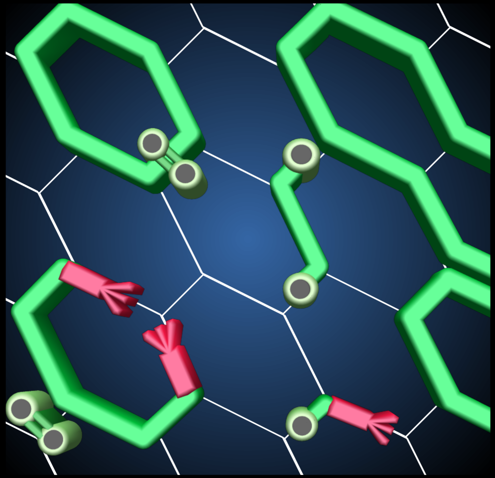
Loop, string, and hadron dynamics in SU(2) Hamiltonian lattice gauge theories



Jesse Stryker

Maryland Center for Fundamental Physics
University of Maryland, College Park

collaboration w/ Indrakshi Raychowdhury (U. Maryland)

APS DNP Fall Meeting

2020/10/31

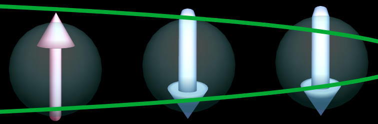
work based on
PRD 101, 114502 (2020)

Quantum simulation for gauge theory

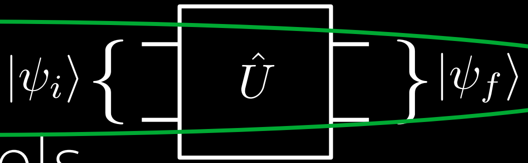
Task: Quantumly simulate QCD degrees of freedom

Need:

- Hilbert space → quantum computer mapping



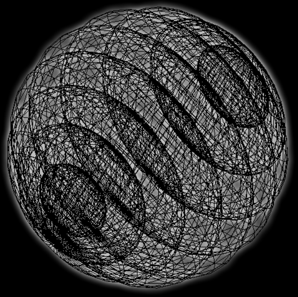
- State preparation
- Time evolution



- Measurement protocols

SU(N) Hamiltonian gauge theory

Algebraic structure of gauge links
- Non-Abelian group, e.g. SU(2)



Lattice gauge transformations:

$$\hat{U}_{n,i} \to \Omega_n \hat{U}_{n,i} \Omega^\dagger_{n+e_i}$$

$\hat{U}_{n,i}$ ~ coordinate operator in SU(2)

"Left" and "right" electric fields to generate the independent left/right rotations on link operators (conjugate momenta)

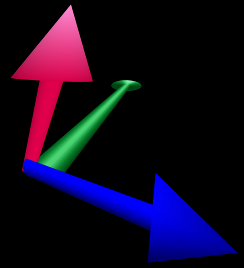
canonical, same-link commutation relations

$$[E^a_{L/R}, E^b_{L/R}] = i f^{abc} E^c_{L/R}$$

$$[E^a_R, U] = U T^a$$

$$[E^a_L, U] = -T^a U$$

Left and right electric fields each have 'colored' components in addition to spatial components



SU(2) 'gluons': 3 components
True gluons: 8 such components

SU(N) Hamiltonian gauge theory

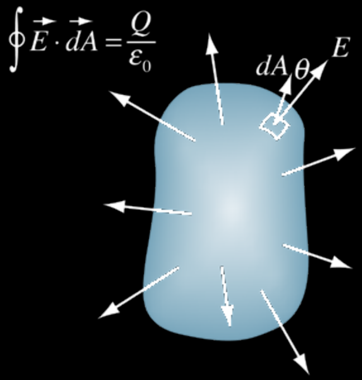
Plus Gauss law constraints

U(1)

$$\nabla \cdot \mathbf{E} - \rho = 0$$

$\hat{\mathcal{G}}_n$

$$\rho = \psi^\dagger \psi$$



SU(N)

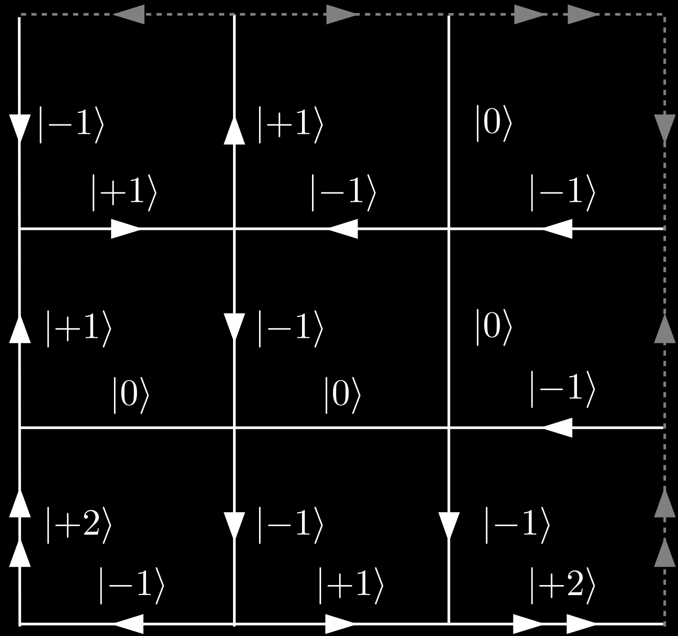
$$\mathbf{D} \cdot \mathbf{E}^a - \rho^a = 0$$

$\hat{\mathcal{G}}_n^a$

$$\rho^a = \psi^\dagger T^a \psi$$

Gauss's law ⟷ gauge invariance

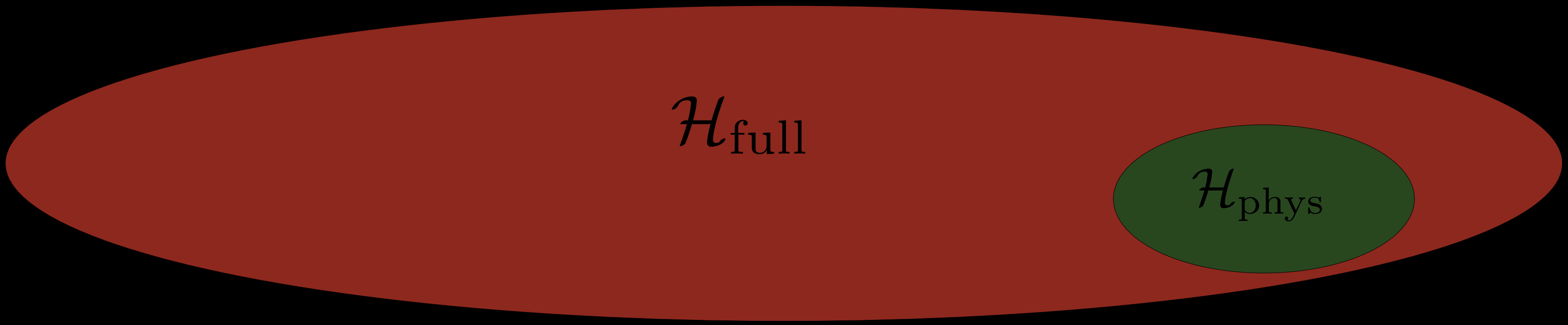
charge conservation



compact U(1)
electric eigenbasis

Implications of basis

- Qubits wasted on unphysical states

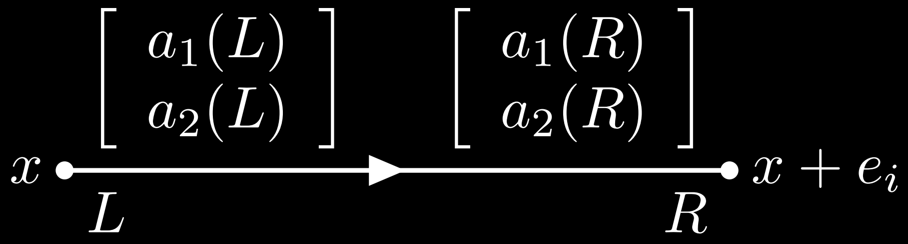


- Non-Abelian constraints mean individual basis states are virtually never allowed by themselves
- Quantum noise will create components along unphysical directions
- Gauge invariance not necessarily respected by algorithms, even for noiseless simulation

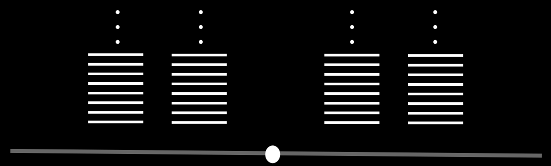
1D: Schwinger bosons + N=2 'quarks'

Step 1: *Start* with **Schwinger boson ("prepotential") formulation**

- Represents gauge field operators using many simple harmonic oscillators *



- One bosonic doublet per end per link
- Gauge Hilbert space → tensor product of SHOs



$$|n_{L,1}, n_{L,2}, n_{R,1}, n_{R,2}\rangle$$
$$n_{L,1} + n_{L,2} = 2j_L$$
$$n_{R,1} + n_{R,2} = 2j_R$$

Gauge transformations:

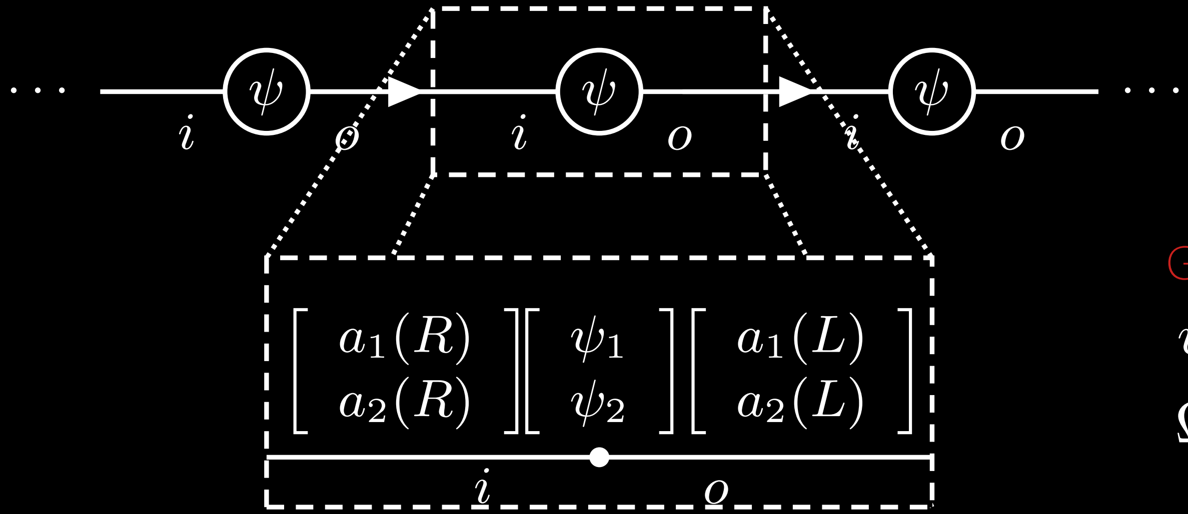
$$a(L) \to \Omega(x) a(L)$$
$$a(R) \to \Omega(x + e_i) a(R)$$
$$\Omega(x), \Omega(x + e_i) \in \mathrm{SU}(2)$$

* Work by: Anishetty, Mathur, Raychowdhury, Sharatchandra, Sreeraj

1D: Schwinger bosons + N=2 ‘quarks’

Step 2: *Add* staggered fermions

- Two-color doublet



- One fermionic doublet per site
- Fock space to characterize lattice

Gauge transformations:

$$\psi(x) \to \Omega(x)\psi(x)$$

$$\Omega(x) \in \mathrm{SU}(2)$$

1D: Schwinger bosons + N=2 'quarks'

Step 3a: *Represent* E, U algebra

$$L_\alpha \equiv \hat{a}^\dagger(L) T_\alpha \hat{a}(L)$$
$$R_\alpha \equiv \hat{a}^\dagger(R) T_\alpha \hat{a}(R)$$

$$\hat{U}(x,i) = \hat{U}_L(x)\hat{U}_R(x+e_i) \ ,$$

$$\hat{U}_L(x,i) = \frac{1}{\sqrt{\mathcal{N}_L + 1}} \begin{pmatrix} \hat{a}_2^\dagger(L) & \hat{a}_1(L) \\ -\hat{a}_1^\dagger(L) & \hat{a}_2(L) \end{pmatrix} \Bigg|_{x,i}$$

$$\hat{U}_R(x,i) = \begin{pmatrix} \hat{a}_1^\dagger(R) & \hat{a}_2^\dagger(R) \\ -\hat{a}_2(R) & \hat{a}_1(R) \end{pmatrix} \frac{1}{\sqrt{\mathcal{N}_R + 1}} \Bigg|_{x,i}$$

3b: Impose "Abelian Gauss law"

$$\mathcal{N}_{L/R} = \hat{a}^\dagger(L/R) \cdot \hat{a}(L/R)$$

$$\mathcal{N}_L(x,i) \left|\text{phys}\right\rangle = \mathcal{N}_R(x+e_i,i) \left|\text{phys}\right\rangle$$

Supplementary constraint
from introducing extra dof's

Loop-string-hadron formulation, 1D

Step 4: *Exploit doublets to make singlets*

Notice:

$$f \to \Omega \cdot f$$
$$(\epsilon f^*) \to \Omega \cdot (\epsilon f^*)$$
$$f = a(L), a(R), \psi$$

So use the doublets and their duals from a site to make *manifestly* **Ω**-*invariant* bilinears (**Ω**†s cancel **Ω**s)

Examples:

$$a(L/R)^\dagger \cdot a(L/R) = a_1^\dagger a_1 + a_2^\dagger a_2$$
$$\psi^\dagger \cdot \psi = \psi_1^\dagger \psi_1 + \psi_2^\dagger \psi_2$$
$$(\epsilon a(L)^*)^\dagger \cdot a(R) = a_1(R) a_2(L) - a_2(R) a_1(L)$$
$$(\epsilon \psi^*)^\dagger \cdot \psi = -(\psi_1 \psi_2 - \psi_2 \psi_1) = 2\psi_2 \psi_1$$

Can form 17 bilinears exactly invariant under **Ω**

These combinations do not "know" a way to violate color charge conservation

Loop-string-hadron formulation, 1D

Key SU(2)-invariant operators:

Can form 17 bilinears exactly invariant under **Ω**

$$\mathcal{L}^{++} = a(R)^\dagger \epsilon\, a(L)$$ "loop" segment creation

$$\mathcal{S}_{\text{in}}^{++} = a(R)^\dagger \epsilon\, \psi^\dagger$$

$$\mathcal{S}_{\text{out}}^{++} = \psi^\dagger \epsilon\, a(L)^\dagger$$

"string" end creation

$$\left(\text{and } \mathcal{H}^{++} = -\tfrac{1}{2}\, \psi^\dagger \epsilon \psi^\dagger\right)$$ "hadron" creation

These combinations do not "know" a way to violate color charge conservation

 $\equiv \mathcal{L}^{++}$  $\equiv \mathcal{S}_{\text{in}}^{++}$  $\equiv \mathcal{S}_{\text{out}}^{++}$

Loop-string-hadron basis, 1D

Use LSH operators to define color-singlet basis

- Take a reference state, 0 flux & 0 fermions
- Act locally with any product of LSH operators
- Result is SU(2)-invariant

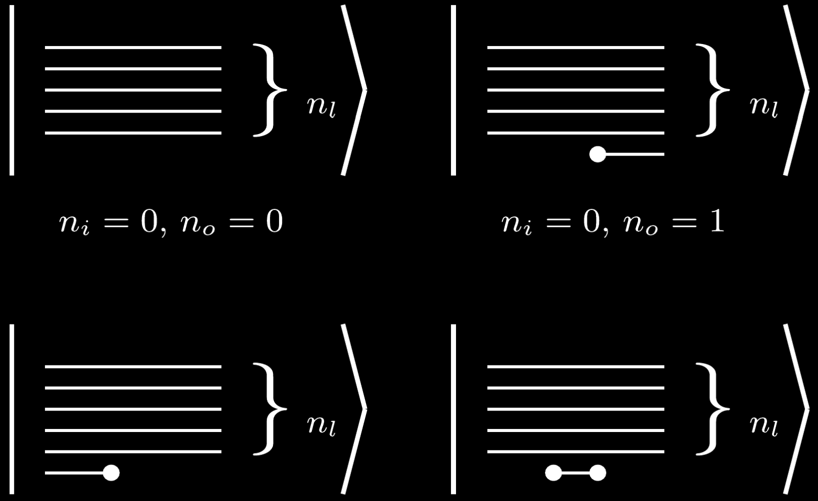
The "catch" of this framework is non-automatic flux conservation *along links.*

$$|n_l, n_i = 0, n_o = 0\rangle \equiv (\mathcal{L}^{++})^{n_l}|0\rangle$$

$$|n_l, n_i = 0, n_o = 1\rangle \equiv (\mathcal{L}^{++})^{n_l}\mathcal{S}_{\text{out}}^{++}|0\rangle$$

$$|n_l, n_i = 1, n_o = 0\rangle \equiv (\mathcal{L}^{++})^{n_l}\mathcal{S}_{\text{in}}^{++}|0\rangle$$

$$|n_l, n_i = 1, n_o = 1\rangle \equiv (\mathcal{L}^{++})^{n_l}\mathcal{H}^{++}|0\rangle$$



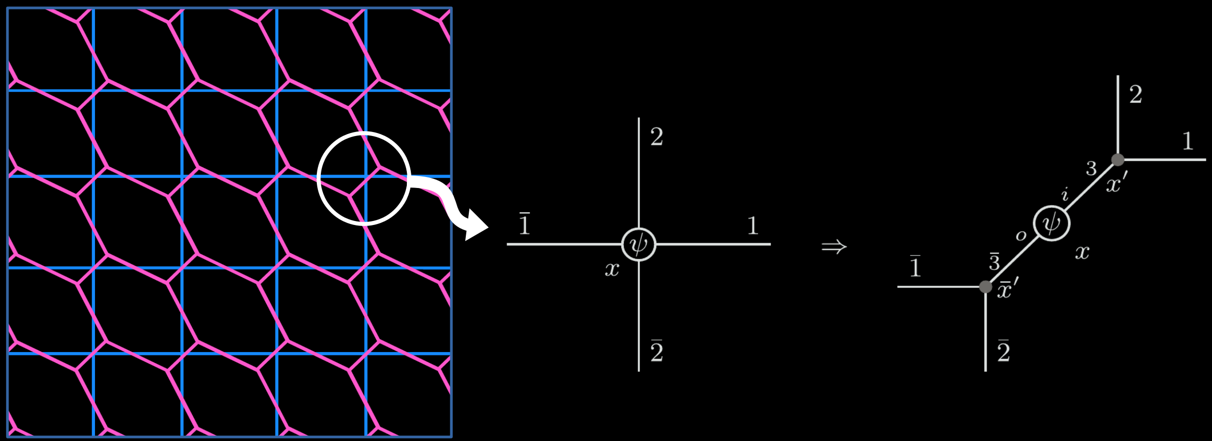
$n_i = 0,\ n_o = 0$ | $n_i = 0,\ n_o = 1$

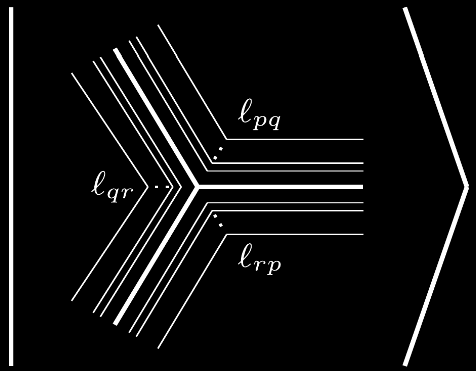
$n_i = 1,\ n_o = 0$ | $n_i = 1,\ n_o = 1$

Loop-string-hadron basis, d>1

Same approach generalizes to d=2,3
once Cartesian lattice is **point split**

d>1 lattices have
- locally-1D "quark sites"
- **"gluonic sites"**





$\sim |\ell_{pq}, \ell_{qr}, \ell_{rp}\rangle$

Gluonic sites: Three bosonic
"loop" quantum numbers

See also: I. Raychowdhury, Eur. Phys. J. C '19

Loop-string-hadron advantages

- SU(2) invariant
- Abelian constraints
- Simple, symmetric quantum numbers
- All operators 1-sparse
- Clebsch-Gordons recast into harmonic oscilllator scaling factors (generalizes to SU(3))
- Reduced gauge redundancy for earliest simulations

Low-dimensional advantages quantified by Davoudi, Raychowdhury, and Shaw (arXiv:2009.11802)
(see next talk by Raychowdhury)

Summary

- Consequences of gauge theory basis are far-reaching
- LSH-formulated SU(2) promising for quantum simulation
  - Exactly implementable gauge invariance ☑ (next talk)
- LSH formulated in d=2,3 ☑
- LSH Hamiltonians made explicit ☑
- SU(3)-generalizable
- **Applications / algorithms now being studied**

Full details at
I. Raychowdhury & JS
PRD 101, 114502 (2020)

This research was supported by the US DOE through the Institute for Nuclear Theory in Seattle, Washington.