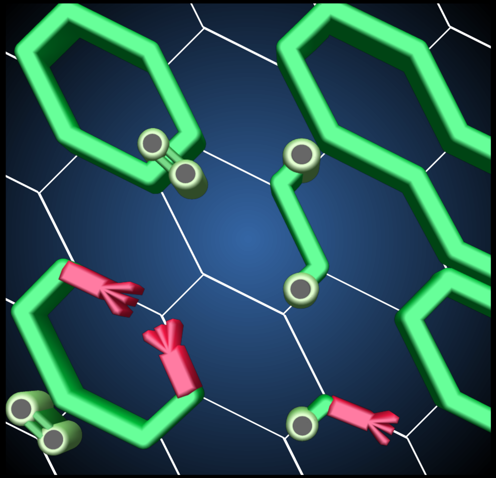
Loop-string-hadron formulation of an SU(3) gauge theory with dynamical quarks



Jesse Stryker
Maryland Center for Fundamental Physics
University of Maryland, College Park

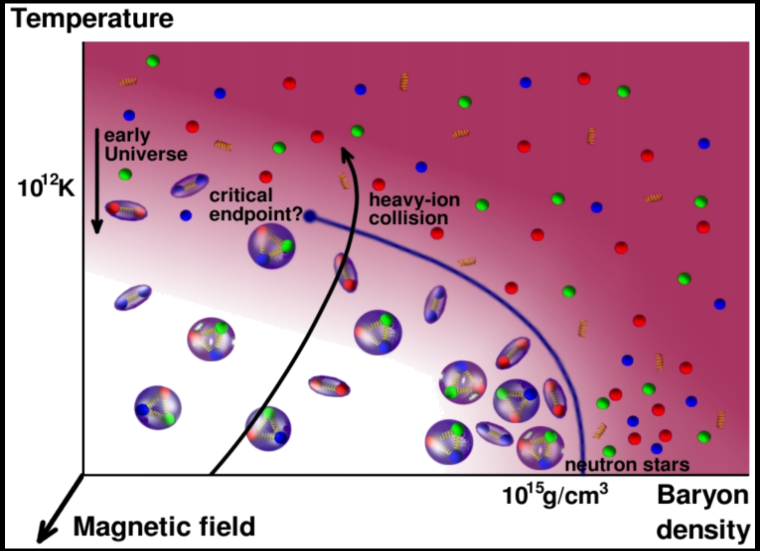
InQubator for Quantum Simulation
virtual seminar
2023-04-05

collaboration w/ Indrakshi Raychowdhury (BITS Pilani, Goa) & Saurabh Kadam (U. Maryland)
arXiv:2212.04490

Motivation

Physics targets:

- Simulation of quantum chromodynamics
  - Hadronization
  - Microscopic understanding of nuclear interactions
- Complete phase diagram of QCD
- Equation of state for nuclear matter



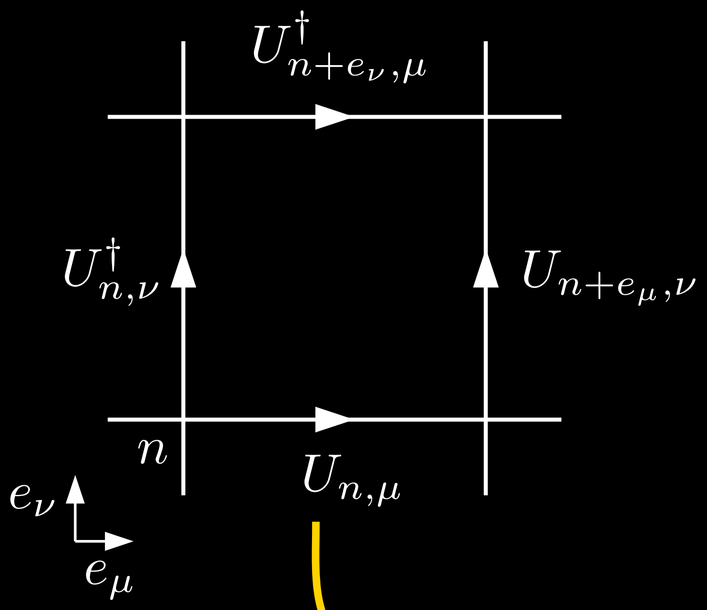
How to make these predictions?

- Nonperturbative problems
  - → **Numerically simulate** QCD degrees of freedom

Conjectured phase diagram credit: G. Endrödi J.Phys.Conf.Ser. 503 (2014) 012009

Traditional lattice field theory

$x^\mu \to a n^\mu$

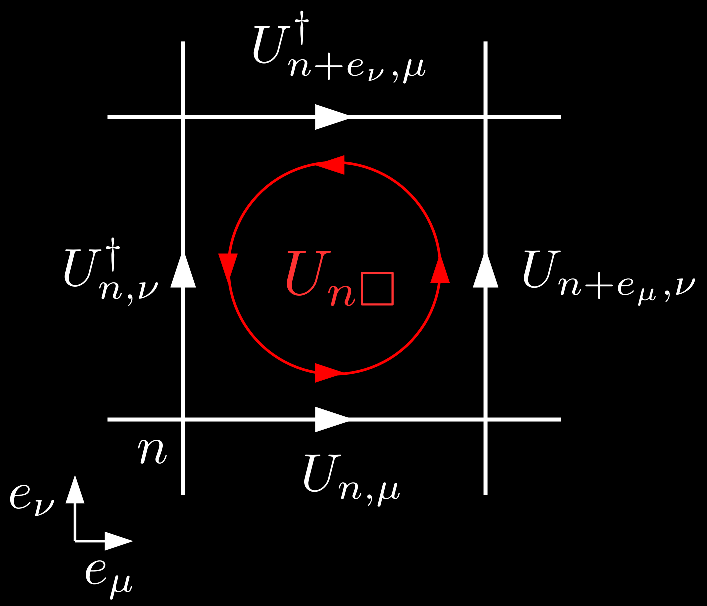


$$\begin{pmatrix} -0.7485 & -0.2744 - 0.6037i \\ 0.2744 - 0.6037i & -0.7485 \end{pmatrix}$$

- Defines a field theory nonperturbatively
- Spacetime discretized with a lattice (e.g. square, cubic, hypercubic)
- Matter particles such as quarks are described “live” on the sites
- Gauge bosons live on oriented links joining sites
- Gauge fields belonging to some Lie group–the “gauge group” *G*

Traditional lattice field theory

$x^\mu \to a n^\mu$



Wilson's gauge action, S_W

"link operator" matrices in gauge group G

$$S_W = -\beta \sum_{n,\mu} \mathrm{tr}\big(\underbrace{U_{n,\mu} U_{n+e_\mu,\nu} U^\dagger_{n+e_\nu,\mu} U^\dagger_{n,\nu}}_{U_\square} + U^\dagger_\square\big)$$

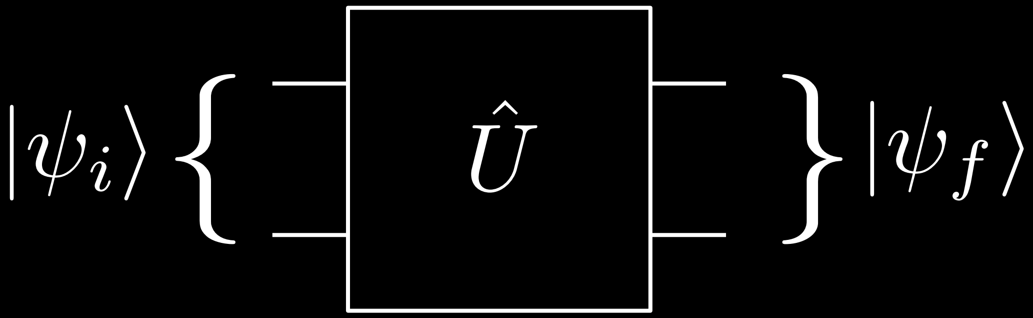
"plaquette" operator

for non-Abelian

- In classical simulations, exp(-S_W) acts like a probably weight for the configuration
- Real-time dynamics and nonzero baryon density both suffer from 'sign problems' in classical simulations

Classical problems.. quantum solutions?

Digital quantum computers:



- Unitary gates: $e^{-it\hat{H}}$ with Hamiltonian of interest
- Want to simulate nonperturbative gauge theory
  - Gauge theory on the lattice
  - Hamiltonian lattice gauge theory
- Has no apparent sign problems

General problem:

How to map a Hilbert space $\mathcal{H}$, and $\hat{H}$, on to qubits & quantum gates?

Outline

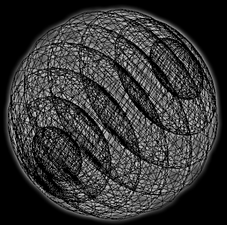
- Hamiltonian lattice gauge theory 101
- Schwinger boson (“prepotential”) reformulation
- SU(2) LSH formulation
- SU(3) generalization of LSH

Hamiltonian lattice gauge theory

- Temporal gauge, continuous-time limit → Kogut-Susskind Hamiltonian formulation
- Gauge fields on spatial links with on-link Hilbert spaces

Phys. Rev. D 11, 395 (1975)

- E.g., SU(2)

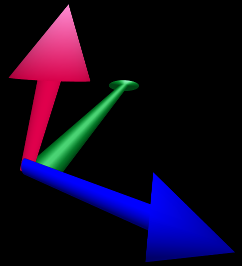


$$\langle g | j, m, m' \rangle = \sqrt{\frac{d_j}{|G|}} D^{(j)}_{m,m'}(g)$$

group-element basis

irrep basis

Left and right electric fields each have color-charge components, in addition to spatial components



$$[\hat{E}^{\alpha}_{L/R}, \hat{E}^{\beta}_{L/R}] = i f^{\alpha\beta\gamma} \hat{E}^{\gamma}_{L/R}$$

$$[\hat{E}^{\alpha}_{R}, \hat{U}_{mm'}] = \left(\hat{U} T^{\alpha}\right)_{mm'}$$

$$[\hat{E}^{\alpha}_{L}, \hat{U}_{mm'}] = -\left(T^{\alpha} \hat{U}\right)_{mm'}$$

canonical commutation relations for a link

Gauge transformations: $\hat{U}_{n,i} \to \Omega_n \hat{U}_{n,i} \Omega^{\dagger}_{n+e_i}$

- Rotations from the left (Ω_n) and right (Ω_{n+ei}) are generated by “left” and “right” electric fields

Hamiltonian lattice gauge theory

Lattice gauge theory Hilbert space structure

- Non-Abelian group, e.g. SU(2)

U adds representations

$$
\begin{aligned}
U_{m,m'} \left|j, M, M'\right\rangle = \\
&C_+(j, m, m', M, M')\times \\
&\quad \times \left|j + 1/2, M + m, M' + m'\right\rangle \\
&+ C_-(j, m, m', M, M')\times \\
&\quad \times \left|j - 1/2, M + m, M' + m'\right\rangle
\end{aligned}
$$

SU(2) example for the
2x2 link operator

Non-Abelian Hamiltonian

$$\hat{H}_E = \frac{g^2}{2} \sum_{n,i} \hat{E}^{\alpha}_{n,i} \hat{E}^{\alpha}_{n,i}$$

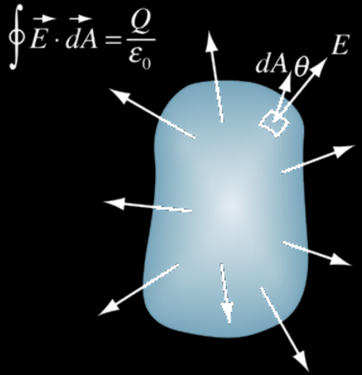
$$\hat{H}_B = -\sum_{n} \frac{1}{2g^2} \mathrm{tr}(\hat{U}_{n,\Box} + \hat{U}^{\dagger}_{n,\Box})$$

Hamiltonian lattice gauge theory

Plus Gauss law constraints

U(1) $\nabla \cdot \mathbf{E} - \rho = 0$

$\hat{\mathcal{G}}_n$ $\rho = \psi^\dagger \psi$

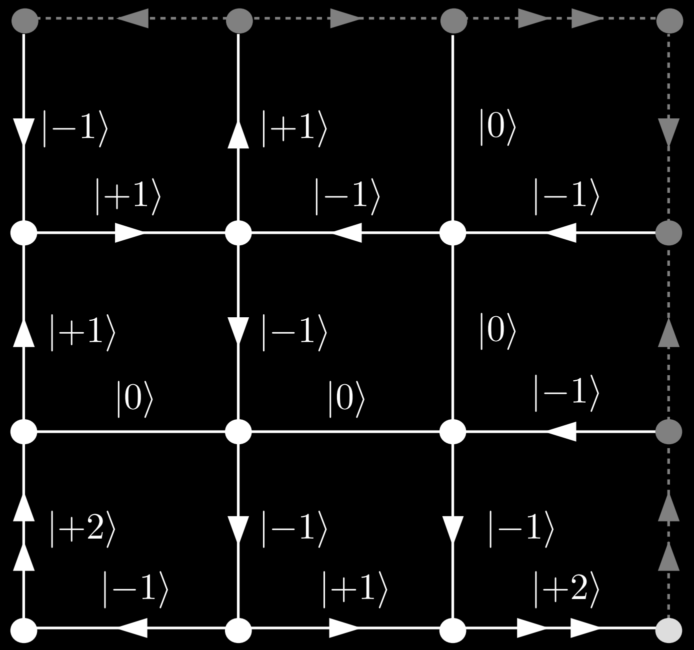


SU(N) $\mathbf{D} \cdot \mathbf{E}^a - \rho^a = 0$

$\hat{\mathcal{G}}_n^a$ $\rho^a = \psi^\dagger T^a \psi$

Gauss's law ⟷ gauge invariance

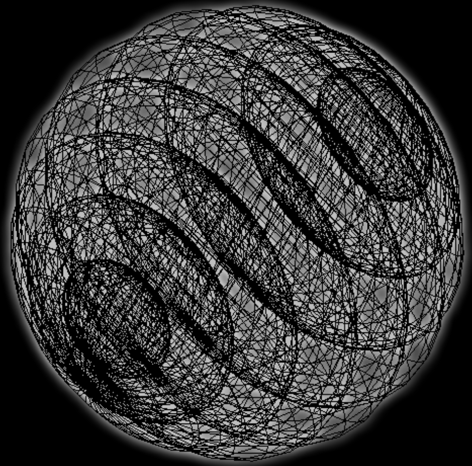
charge conservation



compact U(1)
electric eigenbasis

Warm-up: SU(2)

- Prototype non-Abelian gauge theory: SU(2), 1+1
- Matter: fundamental ‘quarks’
- Goal: Construct a Hamiltonian that has gauge invariance built into it and useful for quantum computation



1D: Schwinger bosons + N=2 'quarks'

Step 1: *Start* with **Schwinger boson ("prepotential") formulation**

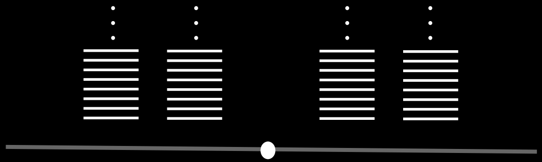
- Represents gauge field operators using many simple harmonic oscillators *

$$\begin{bmatrix} a_1(L) \\ a_2(L) \end{bmatrix} \qquad \begin{bmatrix} a_1(R) \\ a_2(R) \end{bmatrix}$$

$$x \xrightarrow{\hspace{6em}} x + e_i$$

$L \qquad\qquad R$

- One bosonic doublet per end per link
  - ➔ Four total oscillators per link
- Gauge Hilbert space → tensor product of SHOs



$$|n_{L,1}, n_{L,2}, n_{R,1}, n_{R,2}\rangle$$
$$n_{L,1} + n_{L,2} = 2j_L$$
$$n_{R,1} + n_{R,2} = 2j_R$$

Gauge transformations:

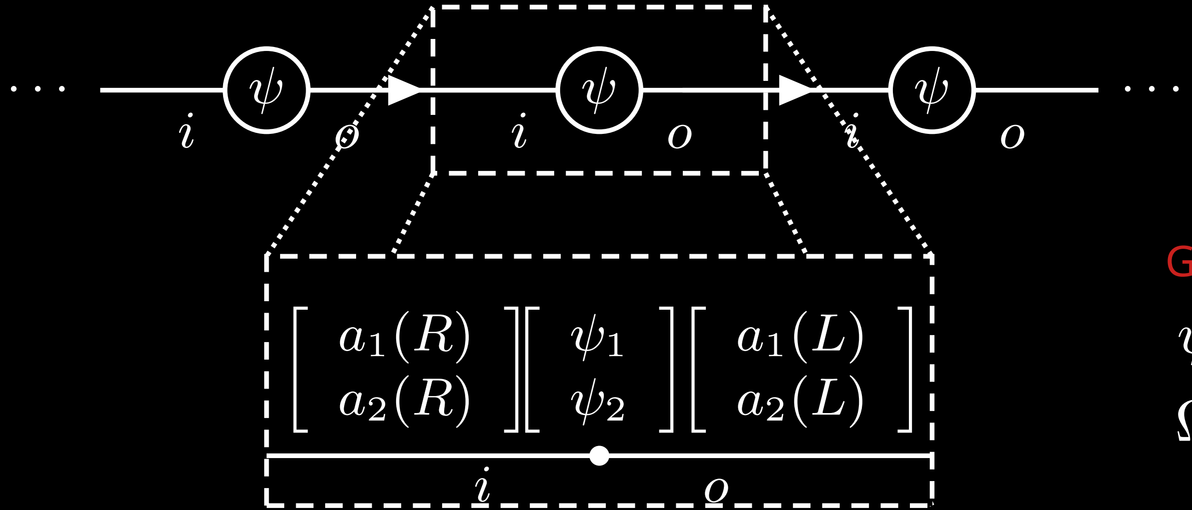
$$a(L) \to \Omega(x) a(L)$$
$$a(R) \to \Omega(x + e_i) a(R)$$
$$\Omega(x), \Omega(x + e_i) \in \mathrm{SU}(2)$$

* Papers by Anishetty, Mathur, Raychowdhury, Sharatchandra

1D: Schwinger bosons + N=2 'quarks'

Step 2: *Add* staggered fermions

- Two-color doublet



- One fermionic doublet per site
- Fock space to characterize lattice

Gauge transformations:

$$\psi(x) \to \Omega(x)\psi(x)$$

$$\Omega(x) \in \mathrm{SU}(2)$$

1D: Schwinger bosons + N=2 ‘quarks’

Step 3a: *Represent* E, U algebra

$$L_\alpha \equiv \hat{a}^\dagger(L) T_\alpha \hat{a}(L)$$

$$R_\alpha \equiv \hat{a}^\dagger(R) T_\alpha \hat{a}(R)$$

$$\hat{U}(x,i) = \hat{U}_L(x)\hat{U}_R(x+e_i) \ ,$$

$$\hat{U}_L(x,i) = \frac{1}{\sqrt{\mathcal{N}_L + 1}} \left(\begin{array}{cc} \hat{a}_2^\dagger(L) & \hat{a}_1(L) \\ -\hat{a}_1^\dagger(L) & \hat{a}_2(L) \end{array} \right) \Bigg|_{x,i}$$

$$\hat{U}_R(x,i) = \left(\begin{array}{cc} \hat{a}_1^\dagger(R) & \hat{a}_2^\dagger(R) \\ -\hat{a}_2(R) & \hat{a}_1(R) \end{array} \right) \frac{1}{\sqrt{\mathcal{N}_R + 1}} \Bigg|_{x,i}$$

3b: Impose “Abelian Gauss law”

$$\mathcal{N}_{L/R} = \hat{a}^\dagger(L/R) \cdot \hat{a}(L/R)$$

$$\mathcal{N}_L(x,i)\,|\mathrm{phys}\rangle = \mathcal{N}_R(x+e_i,i)\,|\mathrm{phys}\rangle$$

Supplementary constraint
from introducing extra dof’s

Loop-string-hadron formulation, SU(2)

Step 4: *Exploit doublets to make singlets*

Notice:
$$f \to \Omega \cdot f$$
$$(\epsilon f^*) \to \Omega \cdot (\epsilon f^*)$$
$$f = a(L), a(R), \psi$$

So use the doublets and their duals from a site to make *manifestly Ω-invariant* bilinears (have $\Omega^\dagger$ and Ω cancel)

<u>Examples:</u>
$$a(L/R)^\dagger \cdot a(L/R) = a_1^\dagger a_1 + a_2^\dagger a_2$$
$$\psi^\dagger \cdot \psi = \psi_1^\dagger \psi_1 + \psi_2^\dagger \psi_2$$
$$(\epsilon a(L)^*)^\dagger \cdot a(R) = a_1(R) a_2(L) - a_2(R) a_1(L)$$
$$(\epsilon \psi^*)^\dagger \cdot \psi = -(\psi_1 \psi_2 - \psi_2 \psi_1) = 2\psi_2 \psi_1$$

In this way we can form 17 bilinears that are exactly invariant under Ω

These special operators do not "know" a way to violate color charge conservation

PRD 101, 114502 (2020)
PRResearch 2, 033039 (2020)

Loop-string-hadron formulation, SU(2)

$$\mathcal{L}^{++} = a(R)^\dagger_\alpha a(L)^\dagger_\beta \epsilon_{\alpha\beta}$$

$$\mathcal{L}^{--} = a(R)_\alpha a(L)_\beta \epsilon_{\alpha\beta} = (\mathcal{L}^{++})^\dagger$$

$$\mathcal{L}^{+-} = a(R)^\dagger_\alpha a(L)_\beta \delta_{\alpha\beta}$$

$$\mathcal{L}^{-+} = a(R)_\alpha a(L)^\dagger_\beta \delta_{\alpha\beta} = (\mathcal{L}^{+-})^\dagger$$

$$\mathcal{S}^{++}_{\text{in}} = a(R)^\dagger_\alpha \psi^\dagger_\beta \epsilon_{\alpha\beta}$$

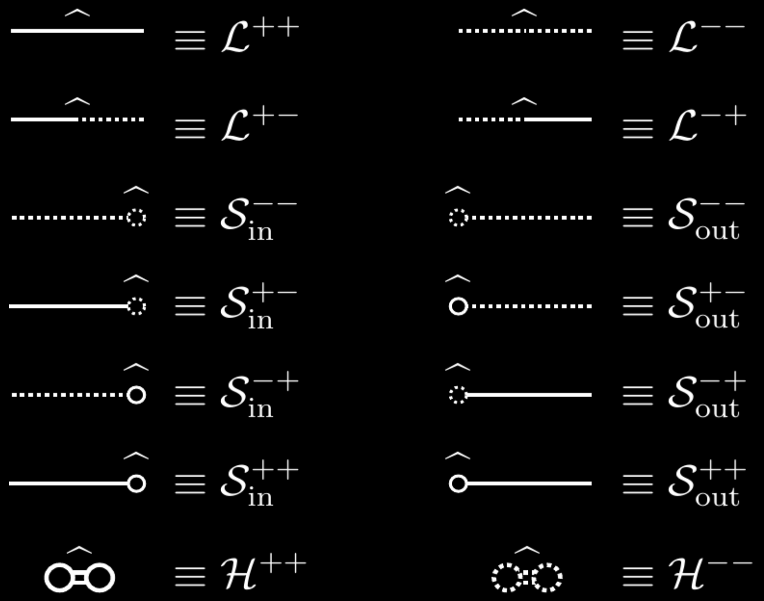
$$\mathcal{S}^{--}_{\text{in}} = a(R)_\alpha \psi_\beta \epsilon_{\alpha\beta} = (\mathcal{S}^{++}_{\text{in}})^\dagger$$

$$\mathcal{S}^{+-}_{\text{in}} = a(R)^\dagger_\alpha \psi_\beta \delta_{\alpha\beta}$$

$$\mathcal{S}^{-+}_{\text{in}} = a(R)_\alpha \psi^\dagger_\beta \delta_{\alpha\beta} = (\mathcal{S}^{+-}_{\text{in}})^\dagger$$

$$\mathcal{H}^{++} = -\frac{1}{2!}\psi^\dagger_\alpha \psi^\dagger_\beta \epsilon_{\alpha\beta}$$

$$\mathcal{H}^{--} = \frac{1}{2!}\psi_\alpha \psi_\beta \epsilon_{\alpha\beta} = (\mathcal{H}^{++})^\dagger$$



In this way we can form 17 bilinears that are exactly invariant under Ω

These special operators do not “know” a way to violate color charge conservation

4 ‘**loop**’ + 4 ‘in **string**’ + 4 ‘out **string**’ + 2 ‘**hadron**’ operators (+ 3 number operators) → “LSH”

LSH formulation, SU(2), 1+1

Physical, SU(2)-invariant interpretations

$\mathcal{L}^{++}(x) \equiv$	Create unit of gauge flux.
$\mathcal{L}^{--}(x) \equiv$	Destroy unit of gauge flux.
$\mathcal{L}^{+-}(x) \equiv$	Change matter-sourced flux direction. $(d > 1)$
$\mathcal{L}^{-+}(x) \equiv$	Change matter-sourced flux direction. $(d > 1)$
$\mathcal{H}^{++}(x) \equiv$	Create a hadron.
$\mathcal{H}^{--}(x) \equiv$	Destroy a hadron.

This *is* the physical intuition for interacting SU(2) excitations

SU(2) = pseudoreal
flux = unoriented

vs

U(1) = complex
flux = oriented

… string operators are less tidy…

LSH formulation, SU(2), 1+1

Found the loop-string-hadron (LSH) operators

- Manifestly SU(2)-invariant
- Transparent physical interpretations
- **Can construct Hamiltonian in terms of them**

"Easy" terms

$$\hat{H}_M \to m_0 \sum_x (-)^x \mathcal{N}_\psi(x)$$

$$\hat{H}_E \to \frac{g_0^2}{4} \sum_x \left[\frac{1}{2} \mathcal{N}_R(x) \left(\frac{1}{2} \mathcal{N}_R(x) + 1 \right) + \frac{1}{2} \mathcal{N}_L(x) \left(\frac{1}{2} \mathcal{N}_L(x) + 1 \right) \right]$$

"Hard" terms

$$\hat{H}_I \to \sum_x \frac{1}{\sqrt{\mathcal{N}_L(x)+1}} \left[\sum_{\sigma=\pm} \mathcal{S}_{\rm out}^{+,\sigma}(x) \mathcal{S}_{\rm in}^{\sigma,-}(x+1) \right] \times \frac{1}{\sqrt{\mathcal{N}_R(x+1)+1}} + \text{H.c.}$$

$$\hat{\psi}^\dagger(x) \hat{U}_L(x) = \frac{1}{\sqrt{\mathcal{N}_L(x)+1}} \begin{pmatrix} \mathcal{S}_{\rm out}^{++}(x), & \mathcal{S}_{\rm out}^{+-}(x) \end{pmatrix},$$

$$\hat{U}_R(x) \hat{\psi}(x) = \begin{pmatrix} \mathcal{S}_{\rm in}^{+-}(x) \\ \mathcal{S}_{\rm in}^{--}(x) \end{pmatrix} \frac{1}{\sqrt{\mathcal{N}_R(x)+1}}.$$

LSH formulation, SU(2), 1+1

LSH operators also define an SU(2)-singlet basis

- Take a reference state, e.g., 0 flux & 0 fermions
- Act locally with any product of LSH operators
- Result is SU(2)-invariant

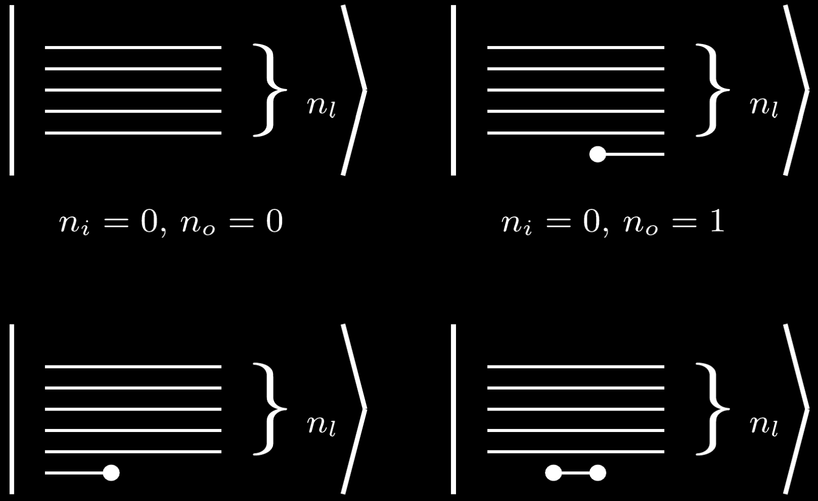
The "catch" of this framework is non-automatic flux conservation *along links.*

$$||n_l, n_i = 0, n_o = 0\rangle \equiv (\mathcal{L}^{++})^{n_l}|0\rangle$$

$$||n_l, n_i = 0, n_o = 1\rangle \equiv (\mathcal{L}^{++})^{n_l}\mathcal{S}^{++}_{\text{out}}|0\rangle$$

$$||n_l, n_i = 1, n_o = 0\rangle \equiv (\mathcal{L}^{++})^{n_l}\mathcal{S}^{++}_{\text{in}}|0\rangle$$

$$||n_l, n_i = 1, n_o = 1\rangle \equiv (\mathcal{L}^{++})^{n_l}\mathcal{H}^{++}|0\rangle$$



$n_i = 0,\ n_o = 0$ $\quad$ $n_i = 0,\ n_o = 1$

$n_i = 1,\ n_o = 0$ $\quad$ $n_i = 1,\ n_o = 1$

$$\mathcal{N}_\psi = \mathcal{N}_i + \mathcal{N}_o$$

$$\mathcal{N}_L = \mathcal{N}_l + \mathcal{N}_o(1 - \mathcal{N}_i)$$

$$\mathcal{N}_R = \mathcal{N}_l + \mathcal{N}_i(1 - \mathcal{N}_o)$$

LSH formulation, SU(2), 1+1

Finale: Compute matrix elements in an orthonormal basis

- All operators ‘factorized’ into diagonal matrices and ‘normalized ladder operators’ (one-sparse, binary matrices)

Loop-string-hadron operator factorizations
$\mathcal{L}^{++} = \Lambda^{+}\sqrt{(\mathcal{N}_l+1)(\mathcal{N}_l+2+(\mathcal{N}_i \oplus \mathcal{N}_o))}$
$\mathcal{L}^{--} = \Lambda^{-}\sqrt{\mathcal{N}_l(\mathcal{N}_l+1+(\mathcal{N}_i \oplus \mathcal{N}_o))}$
$\mathcal{L}^{+-} = -\chi_i^{\dagger}\chi_o$
$\mathcal{L}^{-+} = \chi_i\chi_o^{\dagger}$
$\mathcal{S}_{\text{in}}^{++} = \chi_i^{\dagger}(\Lambda^{+})^{\mathcal{N}_o}\sqrt{\mathcal{N}_l+2-\mathcal{N}_o}$
$\mathcal{S}_{\text{in}}^{--} = \chi_i(\Lambda^{-})^{\mathcal{N}_o}\sqrt{\mathcal{N}_l+2(1-\mathcal{N}_o)}$
$\mathcal{S}_{\text{out}}^{++} = \chi_o^{\dagger}(\Lambda^{+})^{\mathcal{N}_i}\sqrt{\mathcal{N}_l+2-\mathcal{N}_i}$
$\mathcal{S}_{\text{out}}^{--} = \chi_o(\Lambda^{-})^{\mathcal{N}_i}\sqrt{\mathcal{N}_l+2(1-\mathcal{N}_i)}$
$\mathcal{S}_{\text{in}}^{-+} = \chi_o^{\dagger}(\Lambda^{-})^{1-\mathcal{N}_i}\sqrt{\mathcal{N}_l+2\mathcal{N}_i}$
$\mathcal{S}_{\text{in}}^{+-} = \chi_o(\Lambda^{+})^{1-\mathcal{N}_i}\sqrt{\mathcal{N}_l+1+\mathcal{N}_i}$
$\mathcal{S}_{\text{out}}^{+-} = \chi_i^{\dagger}(\Lambda^{-})^{1-\mathcal{N}_o}\sqrt{\mathcal{N}_l+2\mathcal{N}_o}$
$\mathcal{S}_{\text{out}}^{-+} = \chi_i(\Lambda^{+})^{1-\mathcal{N}_o}\sqrt{\mathcal{N}_l+1+\mathcal{N}_o}$
$\mathcal{H}^{++} = \chi_i^{\dagger}\chi_o^{\dagger}$
$\mathcal{H}^{--} = -\chi_i\chi_o$

$$\langle n_l', n_i', n_o' | \Lambda^{\pm} | n_l, n_i, n_o \rangle = \delta_{n_l', n_l \pm 1}\delta_{n_i', n_i}\delta_{n_o', n_o}$$

$$\{\chi_{q'}, \chi_q\} = \{\chi_{q'}^{\dagger}, \chi_q^{\dagger}\} = 0$$

$$\{\chi_{q'}, \chi_q^{\dagger}\} = \delta_{q'q} \qquad (q = i, o)$$

SU(2) LSH & quantum computation

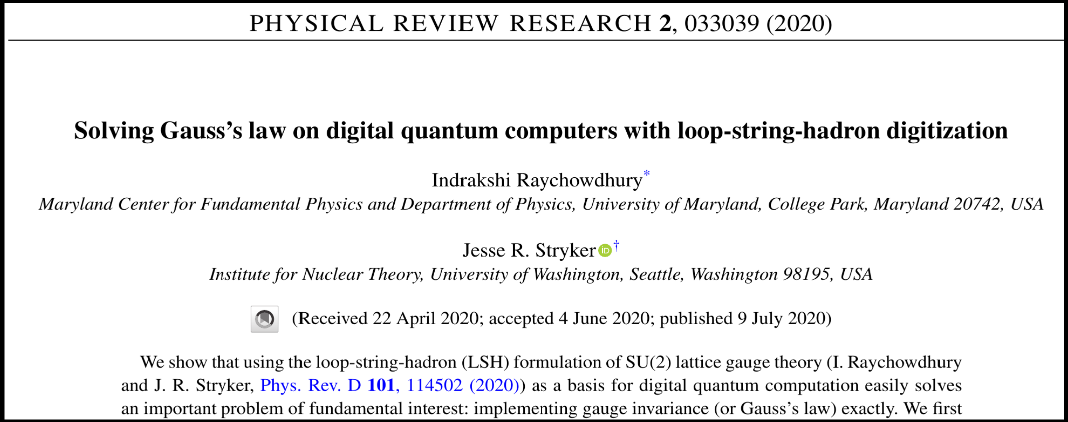
Hamiltonian in operator-factorized form is the input for developing simulation algorithms

Advantages

- All constraints are *Abelian*
  - Simultaneously diagonalizable
  - LSH basis states are individually definitely allowed or definitely unallowed, unlike other formulations
- Hilbert space is structure is far simpler than |jmm'> states
- Hamiltonian structure looks more similar to U(1)
- Clebsch-Gordons recast as SHO scaling factors
- First SU(2) physicality quantum circuits constructed (Raychowdhury & JS 2020)

SU(2) LSH & quantum computation

- Circuits for LSH constraints, in any number of dimensions, are worked out in detail
- Speedups likely needed to make possible in NISQ era


PHYSICAL REVIEW RESEARCH **2**, 033039 (2020)

Solving Gauss's law on digital quantum computers with loop-string-hadron digitization

Indrakshi Raychowdhury*
Maryland Center for Fundamental Physics and Department of Physics, University of Maryland, College Park, Maryland 20742, USA

Jesse R. Stryker†
Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195, USA

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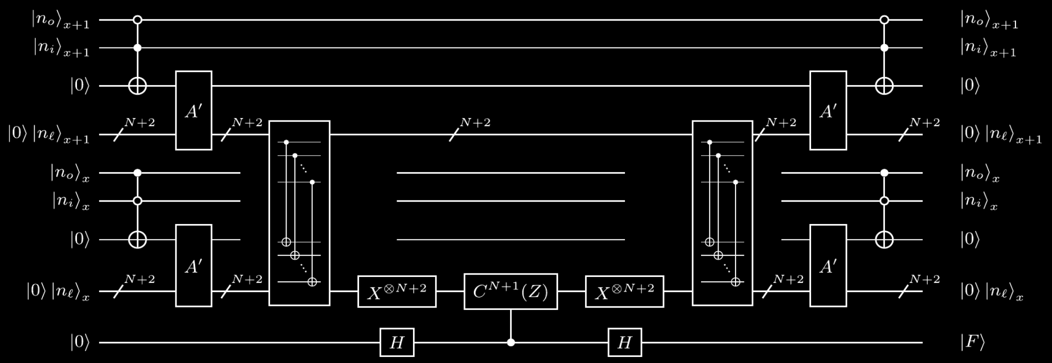
We show that using the loop-string-hadron (LSH) formulation of SU(2) lattice gauge theory (I. Raychowdhury and J. R. Stryker, Phys. Rev. D **101**, 114502 (2020)) as a basis for digital quantum computation easily solves an important problem of fundamental interest: implementing gauge invariance (or Gauss's law) exactly. We first

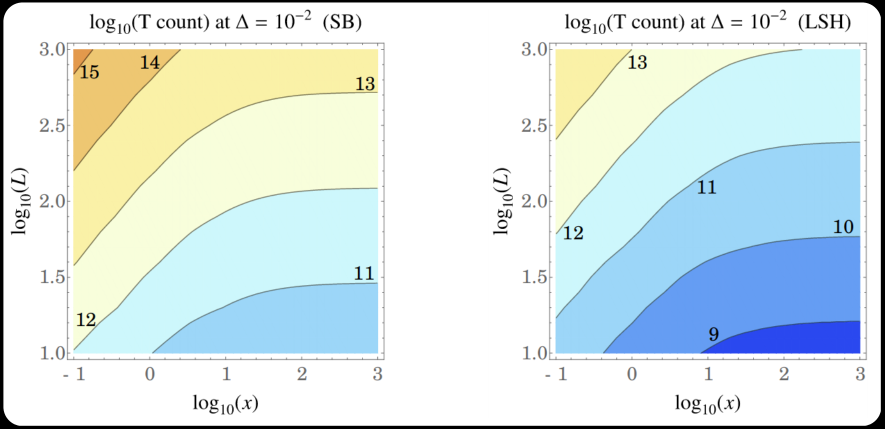
Other LSH shortcomings:

- H_B in d>1 has **many** terms
- Can cost more qubits in d>1



SU(2) LSH vs Schwinger bosons

							Schwinger bosons		LSH	
x	η	L	t/a_s	Δ	$\alpha_{\text{Trot.}}$	$\alpha_{\text{Newt.}}$	Qubits	T gates	Qubits	T gates
1	4	100	1	0.01	90%	9%	2626	8.19713×10^{11}	1319	3.91817×10^{10}
1	4	100	1	0.001	90%	9%	2704	3.09951×10^{12}	1397	1.5172×10^{11}
1	4	100	10	0.01	90%	9%	2704	3.0993×10^{13}	1397	1.51643×10^{12}
1	4	100	10	0.001	90%	9%	2808	1.2146×10^{14}	1475	5.76229×10^{12}
1	4	1000	1	0.01	90%	9%	18904	3.12769×10^{13}	6797	1.53099×10^{12}
1	4	1000	1	0.001	90%	9%	19008	1.22564×10^{14}	6875	5.81562×10^{12}
1	4	1000	10	0.01	90%	9%	19008	1.22564×10^{15}	6875	5.81468×10^{13}
1	4	1000	10	0.001	90%	9%	19086	4.48657×10^{15}	6979	2.29217×10^{14}
1	8	100	1	0.01	90%	9%	4398	5.79224×10^{12}	1807	2.72735×10^{11}
1	8	100	1	0.001	90%	9%	4476	2.1482×10^{13}	1885	1.03709×10^{12}
1	8	100	10	0.01	90%	9%	4476	2.14816×10^{14}	1885	1.03705×10^{13}
1	8	100	10	0.001	90%	9%	4580	8.22615×10^{14}	1963	3.87886×10^{13}
1	8	1000	1	0.01	90%	9%	35076	2.16773×10^{14}	10885	1.04652×10^{13}
1	8	1000	1	0.001	90%	9%	35180	8.30098×10^{14}	10963	3.91414×10^{13}
1	8	1000	10	0.01	90%	9%	35180	8.30094×10^{15}	10963	3.91412×10^{14}
1	8	1000	10	0.001	90%	9%	35258	2.99214×10^{16}	11067	1.5154×10^{15}



T-gate costs at fixed m/g=1. Other simulation parameters not explicitly shown are $\eta = 8$, $t/a_s = 1$, $\alpha_{\text{Trot.}} = 90\%$, $\alpha_{\text{Newt.}} = 9\%$, and $\alpha_{\text{synth.}} = 1\%$.

Z. Davoudi, A.F. Shaw, & JS
arXiv:2212.14030

~20x T gate reduction with LSH

SU(3) Schwinger bosons

- SU(2): Arbitrary irrep *j* constructible by tensor-producting enough spin-1/2's → One doublet $a^\dagger = \begin{pmatrix} a_1^\dagger \\ a_2^\dagger \end{pmatrix}$ to construct all |*j*,*m*> states

- In SU(3): Arbitrary irrep (P,Q) constructible by tensor products of one 3 and one 3* → $a^\dagger = \begin{pmatrix} a_1^\dagger \\ a_2^\dagger \\ a_3^\dagger \end{pmatrix}, \quad b^\dagger = \begin{pmatrix} b^{\dagger 1} \\ b^{\dagger 2} \\ b^{\dagger 3} \end{pmatrix}$

- Ex: 3 $\quad |1,0\rangle_\alpha = a_\alpha^\dagger |\Omega\rangle$

- Ex: 3* $\quad |0,1\rangle^\beta = b^{\dagger\beta} |\Omega\rangle$

SU(3) Schwinger bosons

- Ex: 8 $\quad a^\dagger_\alpha b^{\dagger\beta}\left|\Omega\right\rangle \in (1,1)?$ $\quad$ *No!...* $3\times 3^* = 8 \oplus 1$ $\quad a^\dagger_\alpha b^{\dagger\alpha}\left|\Omega\right\rangle \in (0,0)$

To be *irreducible*, the rep should be *traceless*

$$\left|1,1\right\rangle^\beta_\alpha \equiv a^\dagger_\alpha b^{\dagger\beta}\left|\Omega\right\rangle - \frac{1}{3}\delta^\beta_\alpha a^\dagger\cdot b^\dagger\left|\Omega\right\rangle, \qquad a^\dagger\cdot b^\dagger \equiv a^\dagger_\gamma b^{\dagger\gamma}$$

- One can generalize above to all states, all irreps, but it is horrible to work with analytically

- Solution: "irreducible Schwinger bosons"

Anishetty, Mathur, & Raychowdhury, J. Math. Phys. 50, 053503 (2009)

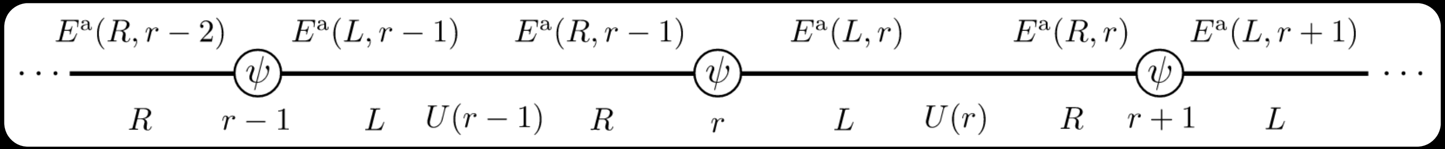
$$A^\dagger_\alpha \equiv a^\dagger_\alpha - \frac{1}{\hat{N}_a+\hat{N}_b+1}(a^\dagger\cdot b^\dagger)b_\alpha, \qquad \hat{N}_a \equiv a^\dagger\cdot a$$

$$B^{\dagger\alpha} \equiv b^{\dagger\alpha} - \frac{1}{\hat{N}_a+\hat{N}_b+1}(a^\dagger\cdot b^\dagger)a^\alpha. \qquad \hat{N}_b \equiv b^\dagger\cdot b$$

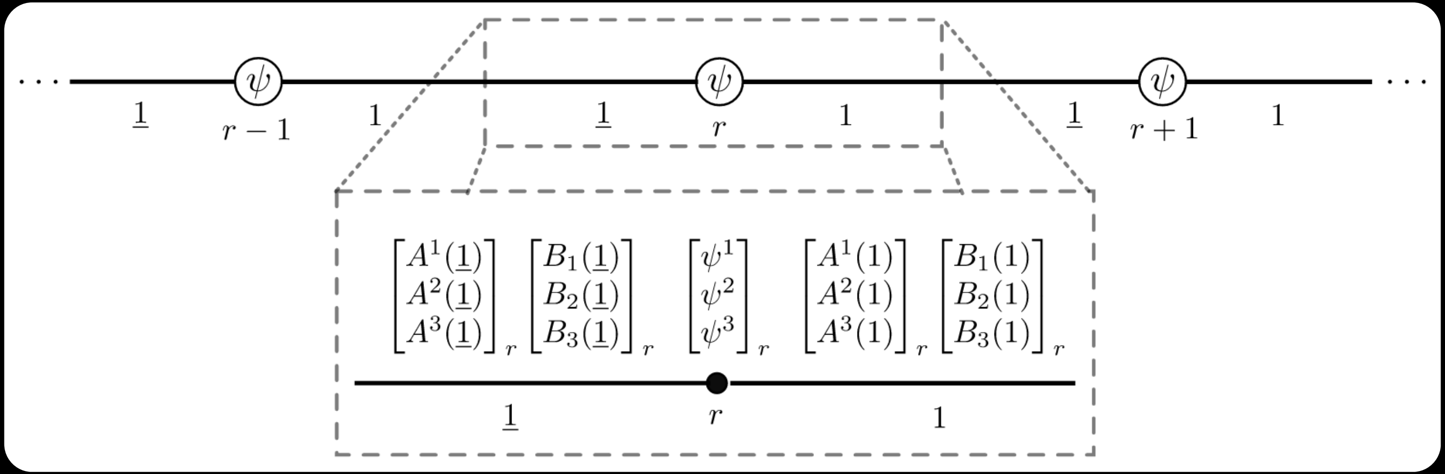
With ISBs: $\left|1,1\right\rangle^\beta_\alpha \equiv A^\dagger_\alpha B^{\dagger\beta}\left|\Omega\right\rangle$

All irrep states have this 'monomial' form

SU(3) Schwinger bosons



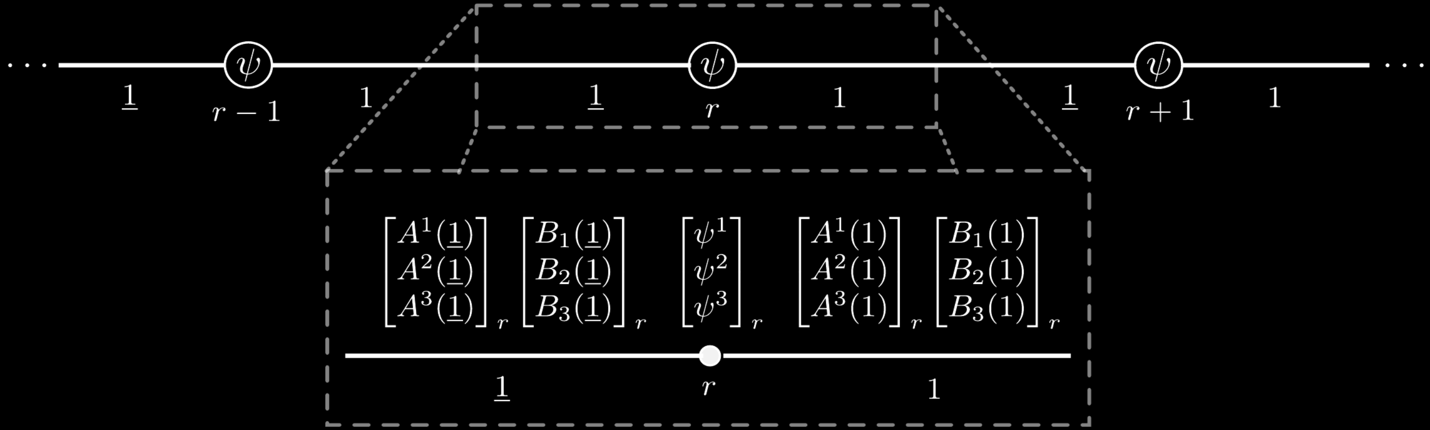
Kogut-Susskind variables



Prepotential variables

SU(3) LSH

Equipped with ISBs, one can now follow the SU(2) procedure



Gauge/matter tensors: $A^\dagger_\alpha,\ B^{\dagger\beta},\ \psi^\dagger_\alpha,\ A^\alpha,\ B_\beta,\ \psi^\alpha$

SU(3)-invariant tensors: $\delta^\beta_\alpha,\ \epsilon^{\alpha\beta\gamma},\ \epsilon_{\alpha\beta\gamma}$

Construct SU(3)-invariants by any allowed contractions

SU(3) LSH

- There are MANY more possible operator contractions with SU(3)…
- The important ones:

$$\hat{N}_\psi \equiv \psi^\dagger \cdot \psi, \quad \hat{P}(\underline{1}) \equiv \hat{N}_A(\underline{1}), \quad \hat{Q}(\underline{1}) \equiv \hat{N}_B(\underline{1}), \quad \hat{P}(1) \equiv \hat{N}_B(1), \quad \hat{Q}(1) \equiv \hat{N}_A(1)$$

$$A(\underline{1})^\dagger \cdot B(1)^\dagger, \quad B(\underline{1})^\dagger \cdot A(1)^\dagger, \qquad \psi^\dagger \cdot B(1)^\dagger, \quad \psi^\dagger \cdot B(\underline{1}), \quad \psi^\dagger \cdot A(1)^\dagger \wedge A(\underline{1})^\dagger$$

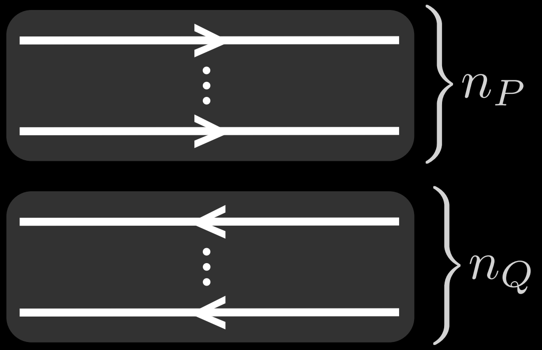
$$\psi^\dagger \cdot B^\dagger(1),\ \psi^\dagger \cdot A(1),\ \psi^\dagger \cdot A^\dagger(1) \wedge B(1),\ \psi^\dagger \cdot B^\dagger(\underline{1}),\ \psi^\dagger \cdot A(\underline{1}),\ \psi^\dagger \cdot A^\dagger(\underline{1}) \wedge B(\underline{1}),\ \text{and H.c.}$$

SU(3) LSH

Hilbert space construction

$$[A^\dagger(\underline{1})\cdot B^\dagger(1)]^{n_P} \rightarrow$$

$$[B^\dagger(\underline{1})\cdot A^\dagger(1)]^{n_Q} \rightarrow$$



$$|n_P, n_Q\rangle \propto |n_P, n_Q; 0,0,0\rangle \rightarrow$$

$$\psi^\dagger \cdot B^\dagger(1)\,|n_P, n_Q\rangle \propto |n_P, n_Q; 0,0,1\rangle \rightarrow$$

$$\psi^\dagger \cdot B^\dagger(\underline{1})\,|n_P, n_Q\rangle \propto |n_P, n_Q; 1,0,0\rangle \rightarrow$$

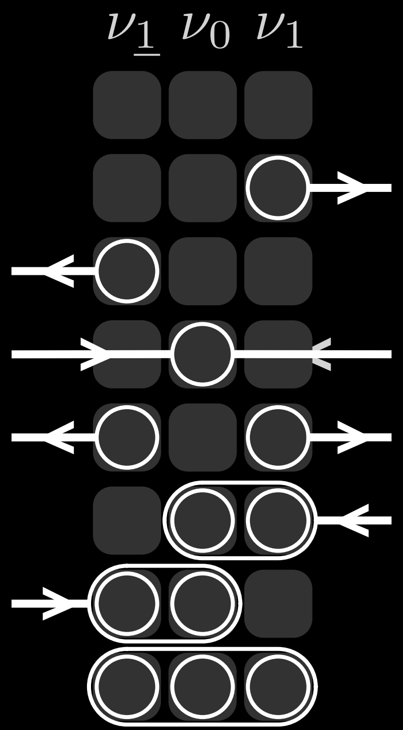
$$\psi^\dagger \cdot A^\dagger(\underline{1}) \wedge A^\dagger(1)\,|n_P, n_Q\rangle \propto |n_P, n_Q; 0,1,0\rangle \rightarrow$$

$$\psi^\dagger \cdot B^\dagger(\underline{1})\psi^\dagger \cdot B^\dagger(1)\,|n_P, n_Q\rangle \propto |n_P, n_Q; 1,0,1\rangle \rightarrow$$

$$\psi^\dagger \cdot \psi^\dagger \wedge A^\dagger(1)\,|n_P, n_Q\rangle \propto |n_P, n_Q; 0,1,1\rangle \rightarrow$$

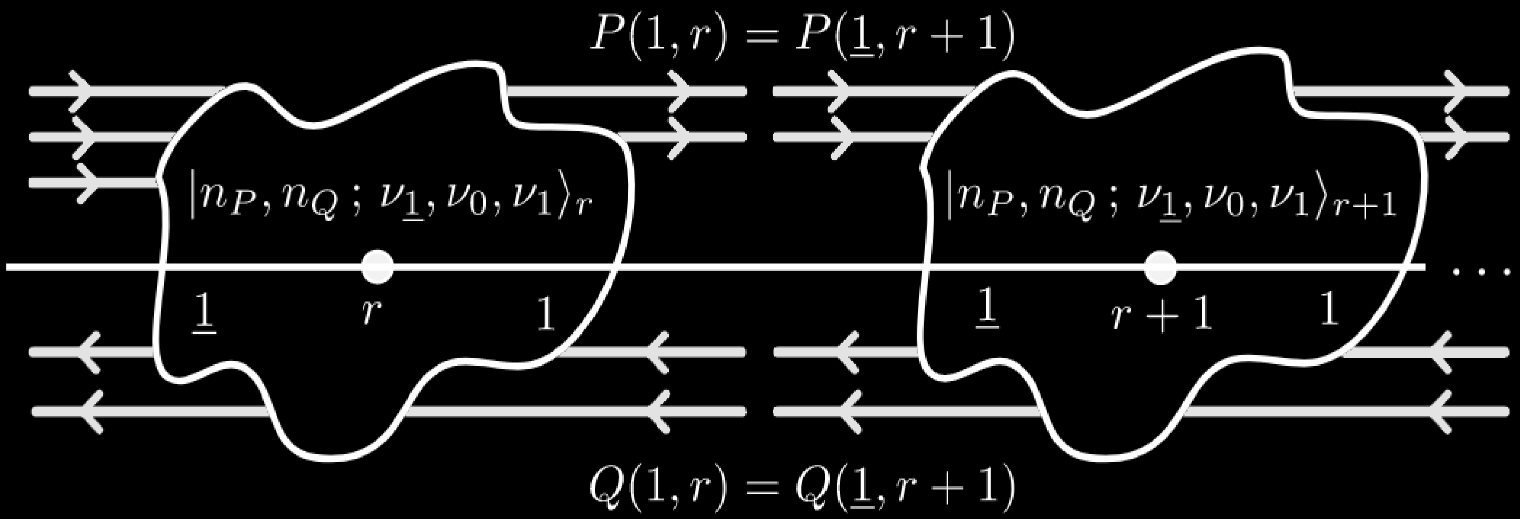
$$\psi^\dagger \cdot \psi^\dagger \wedge A^\dagger(\underline{1})\,|n_P, n_Q\rangle \propto |n_P, n_Q; 1,1,0\rangle \rightarrow$$

$$\psi^\dagger \cdot \psi^\dagger \wedge \psi^\dagger\,|n_P, n_Q\rangle \propto |n_P, n_Q; 1,1,1\rangle \rightarrow$$



SU(3) LSH

Hilbert space construction



Two Abelian constraints per link

SU(3) LSH

$$H_M = \sum_{r=1}^{N} H_M(r) \equiv \mu \sum_{r=1}^{N} (-1)^r \left(\hat{\nu}_{\underline{1}}(r) + \hat{\nu}_0(r) + \hat{\nu}_1(r)\right),$$

$$H_E = \sum_{r=1}^{N'} H_E(r) \equiv \sum_{r=1}^{N'} \frac{1}{3}\left(\hat{P}(1,r)^2 + \hat{Q}(1,r)^2 + \hat{P}(1,r)\hat{Q}(1,r)\right) + \hat{P}(1,r) + \hat{Q}(1,r),$$

$$\begin{aligned}
H_I = \sum_{r=1}^{N'} H_I(r) \equiv \sum_r x & \left[\hat{\chi}_1^\dagger (\hat{\Gamma}_P^\dagger)^{\hat{\nu}_0} \sqrt{1-\hat{\nu}_0/(\hat{n}_P+2)} \sqrt{1-\hat{\nu}_{\underline{1}}/(\hat{n}_P+\hat{n}_Q+3)}\right]_r \\
& \otimes \left[\sqrt{1+\hat{\nu}_0/(\hat{n}_P+1)} \sqrt{1+\hat{\nu}_{\underline{1}}/(\hat{n}_P+\hat{n}_Q+2)}\, \hat{\chi}_1 (\hat{\Gamma}_P^\dagger)^{1-\hat{\nu}_0}\right]_{r+1} \\
+ x & \left[\hat{\chi}_{\underline{1}}^\dagger (\hat{\Gamma}_Q)^{1-\hat{\nu}_0} \sqrt{1+\hat{\nu}_0/(\hat{n}_Q+1)} \sqrt{1+\hat{\nu}_1/(\hat{n}_P+\hat{n}_Q+2)}\right]_r \\
& \otimes \left[\sqrt{1-\hat{\nu}_0/(\hat{n}_Q+2)} \sqrt{1-\hat{\nu}_1/(\hat{n}_P+\hat{n}_Q+3)}\, \hat{\chi}_{\underline{1}} (\hat{\Gamma}_Q)^{\hat{\nu}_0}\right]_{r+1} \\
+ x & \left[\hat{\chi}_0^\dagger (\hat{\Gamma}_P)^{1-\hat{\nu}_1} (\hat{\Gamma}_Q^\dagger)^{\hat{\nu}_{\underline{1}}} \sqrt{1+\hat{\nu}_1/(\hat{n}_P+1)} \sqrt{1-\hat{\nu}_{\underline{1}}/(\hat{n}_Q+2)}\right]_r \\
& \otimes \left[\sqrt{1-\hat{\nu}_1/(\hat{n}_P+2)} \sqrt{1+\hat{\nu}_{\underline{1}}/(\hat{n}_Q+1)}\, \hat{\chi}_0 (\hat{\Gamma}_P)^{\hat{\nu}_1} (\hat{\Gamma}_Q^\dagger)^{1-\hat{\nu}_{\underline{1}}}\right]_{r+1} + \text{H.c.}
\end{aligned}$$

SU(3) LSH

SU(2)

$$H_I(r) \equiv x \left[\hat{\chi}_o^\dagger (\hat{\Gamma}_\ell^+)^{\hat{\nu}_i} \sqrt{1 - \hat{\nu}_i/(\hat{n}_\ell + 2)} \right]_r \otimes \left[\sqrt{1 + \hat{\nu}_i/(\hat{n}_\ell + 1)} \hat{\chi}_o (\hat{\Gamma}_\ell^+)^{1-\hat{\nu}_i} \right]_{r+1}$$

$$+ x \left[\hat{\chi}_i^\dagger (\hat{\Gamma}_\ell^-)^{1-\hat{\nu}_o} \sqrt{1 + \hat{\nu}_o/(\hat{n}_\ell + 1)} \right]_r \otimes \left[\sqrt{1 - \hat{\nu}_o/(\hat{n}_\ell + 2)} \hat{\chi}_i (\hat{\Gamma}_\ell^-)^{\hat{\nu}_o} \right]_{r+1} + \text{H.c.}$$

2 terms instead of 8

SU(3)

$$H_I = \sum_{r=1}^{N'} H_I(r) \equiv \sum_r x \left[\hat{\chi}_1^\dagger (\hat{\Gamma}_P^\dagger)^{\hat{\nu}_0} \sqrt{1 - \hat{\nu}_0/(\hat{n}_P + 2)} \sqrt{1 - \hat{\nu}_{\underline{1}}/(\hat{n}_P + \hat{n}_Q + 3)} \right]_r$$

$$\otimes \left[\sqrt{1 + \hat{\nu}_0/(\hat{n}_P + 1)} \sqrt{1 + \hat{\nu}_{\underline{1}}/(\hat{n}_P + \hat{n}_Q + 2)}\, \hat{\chi}_1 (\hat{\Gamma}_P^\dagger)^{1-\hat{\nu}_0} \right]_{r+1}$$

$$+ x \left[\hat{\chi}_{\underline{1}}^\dagger (\hat{\Gamma}_Q)^{1-\hat{\nu}_0} \sqrt{1 + \hat{\nu}_0/(\hat{n}_Q + 1)} \sqrt{1 + \hat{\nu}_1/(\hat{n}_P + \hat{n}_Q + 2)} \right]_r$$

$$\otimes \left[\sqrt{1 - \hat{\nu}_0/(\hat{n}_Q + 2)} \sqrt{1 - \hat{\nu}_1/(\hat{n}_P + \hat{n}_Q + 3)}\, \hat{\chi}_{\underline{1}} (\hat{\Gamma}_Q)^{\hat{\nu}_0} \right]_{r+1}$$

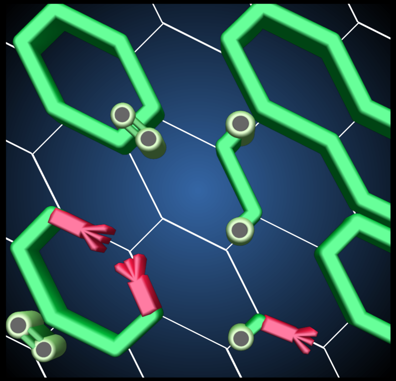
$$+ x \left[\hat{\chi}_0^\dagger (\hat{\Gamma}_P)^{1-\hat{\nu}_1} (\hat{\Gamma}_Q^\dagger)^{\hat{\nu}_{\underline{1}}} \sqrt{1 + \hat{\nu}_1/(\hat{n}_P + 1)} \sqrt{1 - \hat{\nu}_{\underline{1}}/(\hat{n}_Q + 2)} \right]_r$$

$$\otimes \left[\sqrt{1 - \hat{\nu}_1/(\hat{n}_P + 2)} \sqrt{1 + \hat{\nu}_{\underline{1}}/(\hat{n}_Q + 1)}\, \hat{\chi}_0 (\hat{\Gamma}_P)^{\hat{\nu}_1} (\hat{\Gamma}_Q^\dagger)^{1-\hat{\nu}_{\underline{1}}} \right]_{r+1} + \text{H.c.}$$

3 terms instead of 27

Summary

- Consequences of gauge theory basis choice are far-reaching
  - Long, interesting road to quantum computing QCD
- LSH-formulated SU(3) is promising for digital simulation
  - Structurally similar to SU(2)
  - Exactly implementable gauge invariance ☑
- Loop-string-hadron Hamiltonian made explicit ☑
- **Algorithms now needed for SU(3)**
- **Multiple space dimensions and multiple flavors too**



Thank you for your attention!

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Extra slides

Loop-string-hadron algebra, SU(2), 1D

	$[\cdot,\mathcal{N}_R]$	$[\cdot,\mathcal{N}_L]$	$[\cdot,\mathcal{N}_\psi]$	$[\cdot,\mathcal{L}^{--}]$	$[\cdot,\mathcal{L}^{-+}]$	$[\cdot,\mathcal{L}^{+-}]$	$[\cdot,\mathcal{L}^{++}]$	$[\cdot,\mathcal{H}^{++}]$	$[\cdot,\mathcal{H}^{--}]$
$[\mathcal{N}_R,\cdot]$	0	0	0	$-\mathcal{L}^{--}$	$-\mathcal{L}^{-+}$	$+\mathcal{L}^{+-}$	$+\mathcal{L}^{++}$	0	0
$[\mathcal{N}_L,\cdot]$	0	0	0	$-\mathcal{L}^{--}$	$+\mathcal{L}^{-+}$	$-\mathcal{L}^{+-}$	$+\mathcal{L}^{++}$	0	0
$[\mathcal{N}_\psi,\cdot]$	0	0	0	0	0	0	0	$2\mathcal{H}^{++}$	$-2\mathcal{H}^{--}$
$[\mathcal{L}^{++},\cdot]$	$-\mathcal{L}^{++}$	$-\mathcal{L}^{++}$	0	$-\mathcal{N}_L-\mathcal{N}_R-2$	0	0	0	0	0
$[\mathcal{L}^{+-},\cdot]$	$-\mathcal{L}^{+-}$	$+\mathcal{L}^{+-}$	0	0	$\mathcal{N}_R-\mathcal{N}_L$	0	0	0	0
$[\mathcal{L}^{-+},\cdot]$	$+\mathcal{L}^{-+}$	$-\mathcal{L}^{-+}$	0	0	0	$\mathcal{N}_L-\mathcal{N}_R$	0	0	0
$[\mathcal{L}^{--},\cdot]$	$+\mathcal{L}^{--}$	$+\mathcal{L}^{--}$	0	0	0	0	$\mathcal{N}_L+\mathcal{N}_R+2$	0	0
$[\mathcal{S}_{\text{in}}^{++},\cdot]$	$-\mathcal{S}_{\text{in}}^{++}$	0	$-\mathcal{S}_{\text{in}}^{++}$	$-\mathcal{S}_{\text{out}}^{+-}$	$+\mathcal{S}_{\text{out}}^{++}$	0	0	0	$-\mathcal{S}_{\text{in}}^{+-}$
$[\mathcal{S}_{\text{in}}^{+-},\cdot]$	$-\mathcal{S}_{\text{in}}^{+-}$	0	$+\mathcal{S}_{\text{in}}^{+-}$	$-\mathcal{S}_{\text{out}}^{--}$	$-\mathcal{S}_{\text{out}}^{-+}$	0	0	$-\mathcal{S}_{\text{in}}^{++}$	0
$[\mathcal{S}_{\text{in}}^{-+},\cdot]$	$+\mathcal{S}_{\text{in}}^{-+}$	0	$-\mathcal{S}_{\text{in}}^{-+}$	0	0	$+\mathcal{S}_{\text{out}}^{+-}$	$+\mathcal{S}_{\text{out}}^{++}$	0	$+\mathcal{S}_{\text{in}}^{--}$
$[\mathcal{S}_{\text{in}}^{--},\cdot]$	$+\mathcal{S}_{\text{in}}^{--}$	0	$+\mathcal{S}_{\text{in}}^{--}$	0	0	$-\mathcal{S}_{\text{out}}^{--}$	$+\mathcal{S}_{\text{out}}^{-+}$	$+\mathcal{S}_{\text{in}}^{-+}$	0
$[\mathcal{S}_{\text{out}}^{++},\cdot]$	0	$-\mathcal{S}_{\text{out}}^{++}$	$-\mathcal{S}_{\text{out}}^{++}$	$-\mathcal{S}_{\text{in}}^{-+}$	0	$+\mathcal{S}_{\text{in}}^{++}$	0	0	$+\mathcal{S}_{\text{out}}^{-+}$
$[\mathcal{S}_{\text{out}}^{-+},\cdot]$	0	$-\mathcal{S}_{\text{out}}^{-+}$	$+\mathcal{S}_{\text{out}}^{-+}$	$-\mathcal{S}_{\text{in}}^{--}$	0	$-\mathcal{S}_{\text{in}}^{+-}$	0	$+\mathcal{S}_{\text{out}}^{++}$	0
$[\mathcal{S}_{\text{out}}^{+-},\cdot]$	0	$+\mathcal{S}_{\text{out}}^{+-}$	$-\mathcal{S}_{\text{out}}^{+-}$	0	$+\mathcal{S}_{\text{in}}^{-+}$	0	$+\mathcal{S}_{\text{in}}^{++}$	0	$-\mathcal{S}_{\text{out}}^{--}$
$[\mathcal{S}_{\text{out}}^{--},\cdot]$	0	$+\mathcal{S}_{\text{out}}^{--}$	$+\mathcal{S}_{\text{out}}^{--}$	0	$-\mathcal{S}_{\text{in}}^{--}$	0	$+\mathcal{S}_{\text{in}}^{+-}$	$-\mathcal{S}_{\text{out}}^{+-}$	0
$[\mathcal{H}^{--},\cdot]$	0	0	$2\mathcal{H}^{--}$	0	0	0	0	$1-\mathcal{N}_\psi$	0
$[\mathcal{H}^{++},\cdot]$	0	0	$-2\mathcal{H}^{++}$	0	0	0	0	0	$\mathcal{N}_\psi-1$

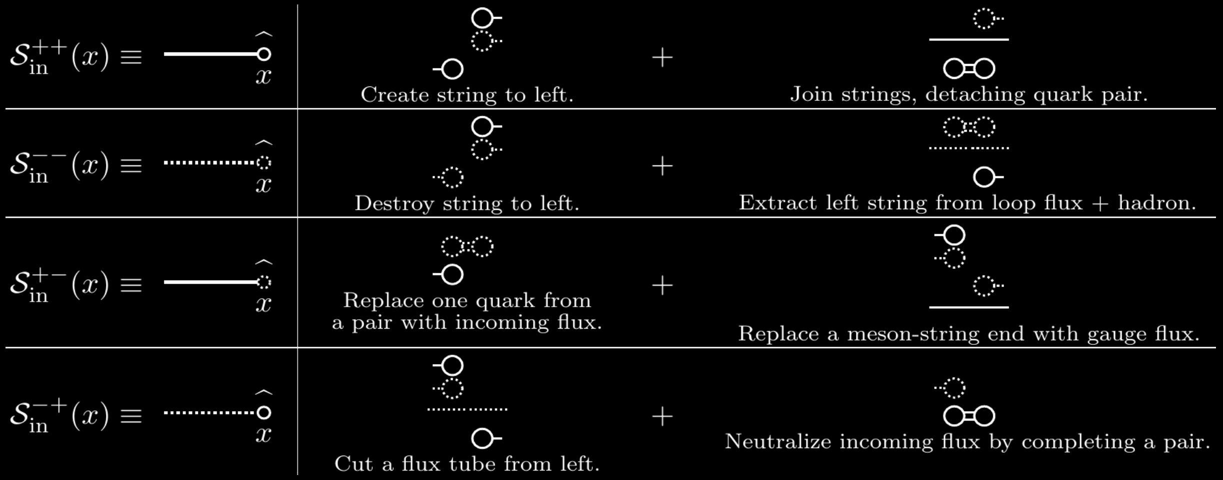
	$\{\cdot,\mathcal{S}_{\text{in}}^{++}\}$	$\{\cdot,\mathcal{S}_{\text{in}}^{+-}\}$	$\{\cdot,\mathcal{S}_{\text{in}}^{-+}\}$	$\{\cdot,\mathcal{S}_{\text{in}}^{--}\}$	$\{\cdot,\mathcal{S}_{\text{out}}^{++}\}$	$\{\cdot,\mathcal{S}_{\text{out}}^{+-}\}$	$\{\cdot,\mathcal{S}_{\text{out}}^{-+}\}$	$\{\cdot,\mathcal{S}_{\text{out}}^{--}\}$
$\{\mathcal{S}_{\text{in}}^{++},\cdot\}$	0	0	$-2\mathcal{H}^{++}$	$2+\mathcal{N}_R-\mathcal{N}_\psi$	0	0	$+\mathcal{L}^{++}$	$-\mathcal{L}^{+-}$
$\{\mathcal{S}_{\text{in}}^{+-},\cdot\}$	0	0	$\mathcal{N}_R+\mathcal{N}_\psi$	$-2\mathcal{H}^{--}$	$+\mathcal{L}^{++}$	$+\mathcal{L}^{+-}$	0	0
$\{\mathcal{S}_{\text{in}}^{-+},\cdot\}$	$-2\mathcal{H}^{++}$	$\mathcal{N}_R+\mathcal{N}_\psi$	0	0	0	0	$+\mathcal{L}^{-+}$	$+\mathcal{L}^{--}$
$\{\mathcal{S}_{\text{in}}^{--},\cdot\}$	$2+\mathcal{N}_R-\mathcal{N}_\psi$	$-2\mathcal{H}^{--}$	0	0	$-\mathcal{L}^{-+}$	$+\mathcal{L}^{--}$	0	0
$\{\mathcal{S}_{\text{out}}^{++},\cdot\}$	0	$+\mathcal{L}^{++}$	0	$-\mathcal{L}^{-+}$	0	$2\mathcal{H}^{++}$	0	$2+\mathcal{N}_L-\mathcal{N}_\psi$
$\{\mathcal{S}_{\text{out}}^{+-},\cdot\}$	0	$+\mathcal{L}^{+-}$	0	$+\mathcal{L}^{--}$	$2\mathcal{H}^{++}$	0	$\mathcal{N}_L+\mathcal{N}_\psi$	0
$\{\mathcal{S}_{\text{out}}^{-+},\cdot\}$	$+\mathcal{L}^{++}$	0	$+\mathcal{L}^{-+}$	0	0	$\mathcal{N}_L+\mathcal{N}_\psi$	0	$2\mathcal{H}^{--}$
$\{\mathcal{S}_{\text{out}}^{--},\cdot\}$	$-\mathcal{L}^{+-}$	0	$+\mathcal{L}^{--}$	0	$2+\mathcal{N}_L-\mathcal{N}_\psi$	0	$2\mathcal{H}^{--}$	0

Loop-string-hadron formulation, SU(2), 1D

String operator physical interpretations:

Each string operator is a sum of two possible actions.

The possible choices project onto orthogonal spaces



Loop-string-hadron formulation, SU(2), 1D

LSH Hamiltonian, 1D, factorized form

$$\hat{H}_E = \frac{g_0^2}{4}\sum_x \left\{ \left[\frac{1}{2}(\mathcal{N}_l + \mathcal{N}_o(1-\mathcal{N}_i))\right]_x \times \left[\frac{1}{2}(\mathcal{N}_l + \mathcal{N}_o(1-\mathcal{N}_i)) + 1\right]_x + \left[\frac{1}{2}(\mathcal{N}_l + \mathcal{N}_i(1-\mathcal{N}_o))\right]_x \times \left[\frac{1}{2}(\mathcal{N}_l + \mathcal{N}_i(1-\mathcal{N}_o)) + 1\right]_x \right\}$$

$$\hat{H}_M = m_0 \sum_x (-)^x (\mathcal{N}_i(x) + \mathcal{N}_o(x))$$

$$\mathcal{S}^{++}_{\text{out}}(x)\mathcal{S}^{+-}_{\text{in}}(x+1) = \left[\chi_o^\dagger\right]_x [\chi_o]_{x+1} \left[(1-\mathcal{N}_i) + \Lambda^+\mathcal{N}_i\right]_x \left[\mathcal{N}_i + \Lambda^+(1-\mathcal{N}_i)\right]_{x+1} \times \left[\sqrt{\mathcal{N}_l - \mathcal{N}_i + 2}\right]_x \left[\sqrt{\mathcal{N}_l - (1-\mathcal{N}_i) + 2}\right]_{x+1},$$

$$\mathcal{S}^{--}_{\text{out}}(x)\mathcal{S}^{-+}_{\text{in}}(x+1) = [\chi_o]_x \left[\chi_o^\dagger\right]_{x+1} \left[(1-\mathcal{N}_i) + \Lambda^-\mathcal{N}_i\right]_x \left[\mathcal{N}_i + \Lambda^-(1-\mathcal{N}_i)\right]_{x+1} \times \left[\sqrt{\mathcal{N}_l + 2(1-\mathcal{N}_i)}\right]_x \left[\sqrt{\mathcal{N}_l + 2\mathcal{N}_i}\right]_{x+1},$$

$$\mathcal{S}^{+-}_{\text{out}}(x)\mathcal{S}^{--}_{\text{in}}(x+1) = \left[\chi_i^\dagger\right]_x [\chi_i]_{x+1} \left[\mathcal{N}_o + \Lambda^-(1-\mathcal{N}_o)\right]_x \left[(1-\mathcal{N}_o) + \Lambda^-\mathcal{N}_o\right]_{x+1} \times \left[\sqrt{\mathcal{N}_l + 2\mathcal{N}_o}\right]_x \left[\sqrt{\mathcal{N}_l + 2(1-\mathcal{N}_o)}\right]_{x+1},$$

$$\mathcal{S}^{-+}_{\text{out}}(x)\mathcal{S}^{++}_{\text{in}}(x+1) = [\chi_i]_x \left[\chi_i^\dagger\right]_{x+1} \left[\mathcal{N}_o + \Lambda^+(1-\mathcal{N}_o)\right]_x \left[(1-\mathcal{N}_o) + \Lambda^+\mathcal{N}_o\right]_{x+1} \times \left[\sqrt{\mathcal{N}_l + \mathcal{N}_o + 1}\right]_x \left[\sqrt{\mathcal{N}_l + (1-\mathcal{N}_o) + 1}\right]_{x+1}.$$