Formal and algorithmic developments for quantum-simulating non-Abelian and higher-dimensonal gauge theories

Jesse Stryker
Maryland Center for Fundamental Physics
University of Maryland, College Park

“Toward Quantum Advantage in High-Energy Physics” workshop
MIAPbP, 2023-04-20

Checklist for quantum simulation of QCD

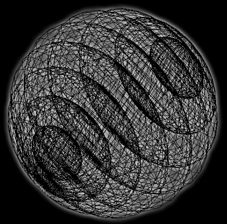
- Digital quantum simulation (DQS) of lattice QCD requires protocols for...
  - initial state preparation
  - time evolution
  - observable measurement
- Error quantification
- Here, lattice QCD means...
  - SU(3) interactions
  - ≥ 2 quark flavors
  - 3D spatial lattice
- First-principles framework: Hamiltonian (non-Abelian) lattice gauge theory

Hamiltonian lattice gauge theory

- Temporal gauge, continuous-time limit → Kogut-Susskind Hamiltonian formulation
- Gauge fields on spatial links with on-link Hilbert spaces

Phys. Rev. D 11, 395 (1975)

- E.g., SU(2)

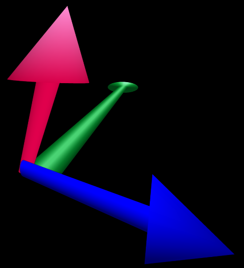


$$\langle g | j, m, m' \rangle = \sqrt{\frac{d_j}{|G|}} D^{(j)}_{m,m'}(g)$$

group-element basis

irrep basis

Left and right electric fields each have color-charge components, in addition to spatial components



$$[\hat{E}^{\alpha}_{L/R}, \hat{E}^{\beta}_{L/R}] = i f^{\alpha\beta\gamma} \hat{E}^{\gamma}_{L/R}$$
$$[\hat{E}^{\alpha}_{R}, \hat{U}_{mm'}] = \left(\hat{U} T^{\alpha}\right)_{mm'}$$
$$[\hat{E}^{\alpha}_{L}, \hat{U}_{mm'}] = -\left(T^{\alpha} \hat{U}\right)_{mm'}$$
$$[\hat{U}_{mm'}, \hat{U}_{ll'}] = [\hat{U}_{mm'}, \hat{U}^{\dagger}_{ll'}] = 0$$

canonical commutation relations for a link

Gauge transformations: $\hat{U}_{n,i} \to \Omega_n \hat{U}_{n,i} \Omega^{\dagger}_{n+e_i}$

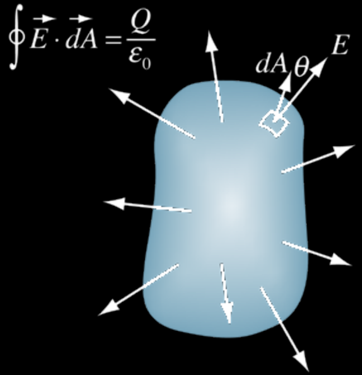
- Rotations from the left (Ω_n) and right (Ω_{n+ei}) are generated by “left” and “right” electric fields

Hamiltonian lattice gauge theory

Plus Gauss law constraints

U(1) $\nabla \cdot \mathbf{E} - \rho = 0$

$\hat{\mathcal{G}}_n$ $\rho = \psi^\dagger \psi$

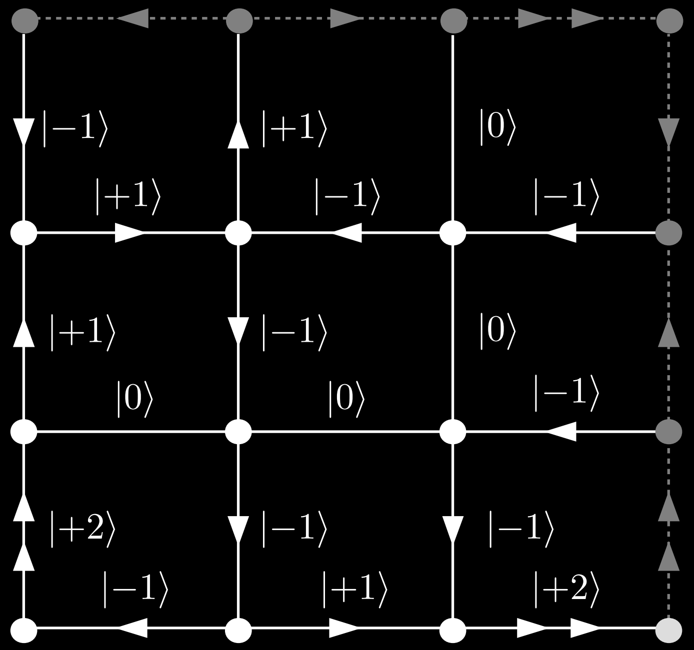


SU(N) $\mathbf{D} \cdot \mathbf{E}^a - \rho^a = 0$

$\hat{\mathcal{G}}_n^a$ $\rho^a = \psi^\dagger T^a \psi$

Gauss's law ⟷ gauge invariance

charge conservation



compact U(1)
electric eigenbasis

Formulations & bases

- Hamiltonian lattice gauge theories seem to enjoy lots of different formulations
- Hamiltonian “formulation” meaning… *
  - set of degrees of freedom - usually local
  - set of fields used to construct Hamiltonian/observables
  - algebraic (commutation) relations
  - constraints
  - (optional truncation scheme)

* my current working definition of formulation; subject to refinement!

Formulations & bases

- Formulation != basis
  - But: Some formulations are often associated with or defined w.r.t. a particular basis
  - Colloquially, different bases are at times called different “formulations” too…
- A formulation isn’t intrinsically tied to a particular Hamiltonian either – different choices are possible!
  - In practice, there usually is an implicit or explicit choice
  - Can’t really do much with a formulation until at least one Hamiltonian has been spelled out
- All bases in use (known to me) are either <u>electric</u> or <u>magnetic</u>

Formulations & bases: Examples

- Kogut-Susskind formulation
  - Irrep/"angular momentum" basis
  Byrnes, Yamamoto, Zohar, Burrello, et al.
  - Group-element basis *Zohar, NuQS collab., et al.*
- Gauge magnets/quantum link models
  Wiese, Chandrasekharan, et al.
- Tensor lattice field theory
  Meurice, Sakai, Unmuth-Yockey, et al.
- Dual/rotor formulations *Kaplan, **JRS**, Haase, Dellantonio, et al., Bauer, Grabowska, Kane*
- Casimir variables / "local-multiplet basis"
  *Klco, Savage, **JRS**, Ciavarella*
- Purely fermionic formulations (1+1D & OBC)
  Muschik, Atas, Zhang, IQuS@UW group, Powell, et al.
- Prepotential/Schwinger boson formulations
  Mathur, Anishetty, Raychowdhury, et al.
- Loop-string-hadron formulation
  *Raychowdhury, **JRS**, Davoudi, Shaw, Dasgupta, Kadam*
- Light-front formulation
  Kreshchuk, Kirby, Love, Yao, et al.
- Qubit models *Chandrasekharan, Singh, et al.*
- *q*-deformed Kogut-Susskind
  Zache, González-Cuadra, Zoller
- Scalar field theory…
  - Harmonic oscillator basis
    Klco & Savage
  - Single-particle basis
    Barata, Mueller, Tarasov, Venugopalan
  - Future gauge-field generalizations??

Choice of basis

Most common basis choice: **Electric/irrep**

Electric-basis pros

- States naturally discretized (for compact Lie groups)
- Gauss's law a function of electric fields
- Natural "UV" truncation scheme
  - Easily translates to truncating operators

Electric-basis cons

- Better-suited to strong coupling (opposite of continuum QCD)
- Many off-diagonal operators in 3+1 Hamiltonian

Electric truncation

- Lie group Hilbert spaces are locally infinite-dimensional
- Digital quantum simulation requires truncations
  - Common choices: Finite subgroups, electric cutoff on irreps

Provably accurate simulation of gauge theories and bosonic systems

Yu Tong[1,2], Victor V. Albert[3], Jarrod R. McClean[1], John Preskill[4,5], and Yuan Su[1,4]
April 4th, 2022

- Tong et al., '22:
  - formal analysis on error in time evolution operator
  - U(1) and SU(2) LGTs considered
  - Find: For fixed error ε and lattice parameters, required electric cutoff grows at worst linearly in time T and polylog($1/\varepsilon$)

Choice of basis

Group-element basis <u>pros</u>

- Link operators are diagonalized
- No Clebsch-Gordon coefficients
- Well-suited for weak-coupling limit

A detail of Spinoza monument in Amsterdam. © Dmitry Feichtner-Kozlov

Group-element basis <u>cons</u>

- Limited number of regular subgroups for SU(N)
  - Limited “resolution” with subgroups
  - 120 elements for SU(2)
  - 1080 for SU(3) [NuQS collab.]
- Sub*sets* generally do not preserve gauge symmetry
- Electric fields become tricky

Choice of basis

- $E^{\alpha}E^{\alpha}$ is Laplace-Beltrami differential operator on the group manifold
- How to define derivatives on a subgroup or discrete subset? How to preserve gauge invariance?
- Only recently has this question been taken up by some groups in the context of quantum simulation

Jakobs, Garofalo, et al.
2304.02322
Mariani, Pradhan, and Ercolessi.
[2301.12224]
Ji, Lamm, and Ju.
Phys. Rev. D 102, 114513 (2020)

Fig. 1 Fibonacci lattices on S_2 with 20 (blue), 100 (orange) and 500 (green) vertices

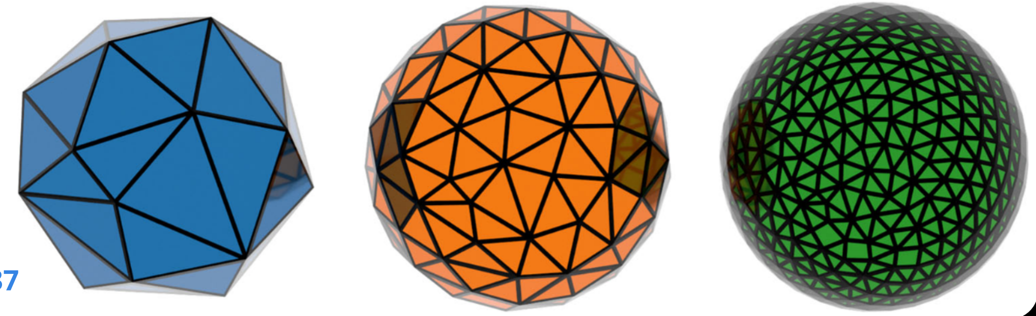
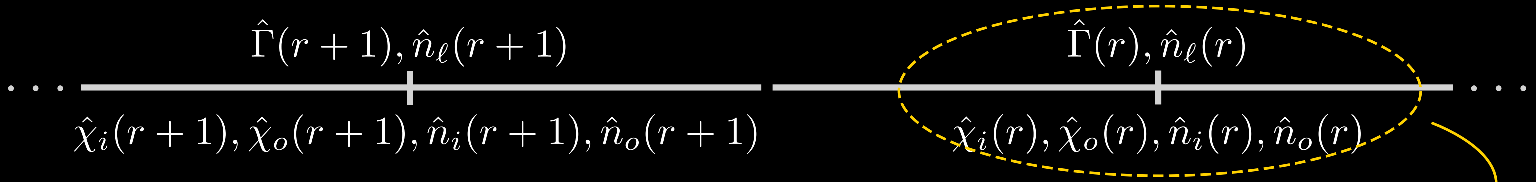


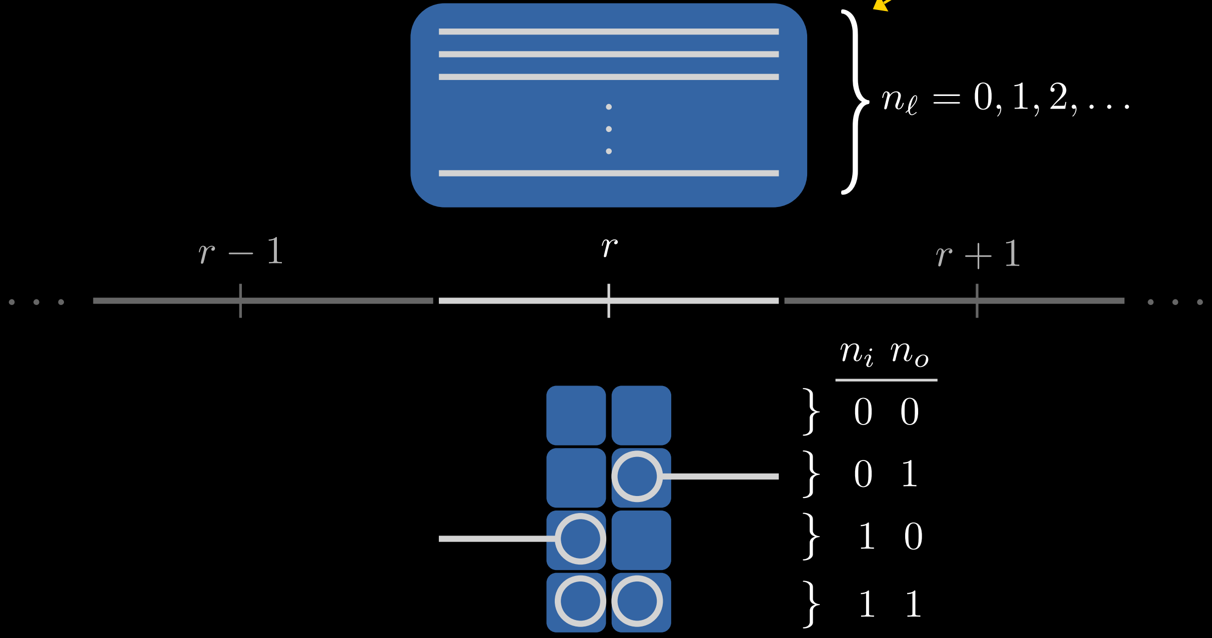
Figure by Hartung, Jakobs, Jansen, Ostmeyer, and Urbach.
Eur. Phys. J. C (2022) 82:237

Loop-string-hadron formulations

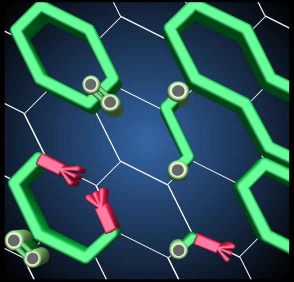
Raychowdhury & Stryker, PRD '20



Loop-string-hadron formulation is derived from Schwinger-boson formulation but uses fewer bosonic DOFs per site. Elementary fields are strictly SU(2) invariant



Loop-string-hadron formulations



LSH states subject to "Abelian Gauss law"

LSH operators define an SU(2)-singlet basis

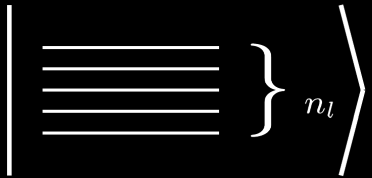
- Take a reference state, e.g., 0 flux & 0 fermions
- Act locally with any product of LSH operators
- Result is SU(2)-invariant

$$\|n_l, n_i = 0, n_o = 0\rangle \equiv (\mathcal{L}^{++})^{n_l}|0\rangle$$

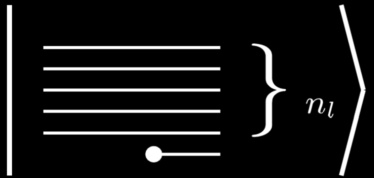
$$\|n_l, n_i = 0, n_o = 1\rangle \equiv (\mathcal{L}^{++})^{n_l}\mathcal{S}^{++}_{\text{out}}|0\rangle$$

$$\|n_l, n_i = 1, n_o = 0\rangle \equiv (\mathcal{L}^{++})^{n_l}\mathcal{S}^{++}_{\text{in}}|0\rangle$$

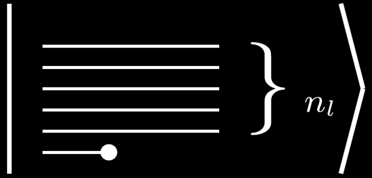
$$\|n_l, n_i = 1, n_o = 1\rangle \equiv (\mathcal{L}^{++})^{n_l}\mathcal{H}^{++}|0\rangle$$



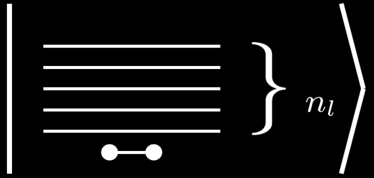
$n_i = 0,\ n_o = 0$



$n_i = 0,\ n_o = 1$



$n_i = 1,\ n_o = 0$



$n_i = 1,\ n_o = 1$

$$\mathcal{N}_\psi = \mathcal{N}_i + \mathcal{N}_o$$

$$\mathcal{N}_L = \mathcal{N}_l + \mathcal{N}_o(1 - \mathcal{N}_i)$$

$$\mathcal{N}_R = \mathcal{N}_l + \mathcal{N}_i(1 - \mathcal{N}_o)$$

SU(2) LSH & quantum computation

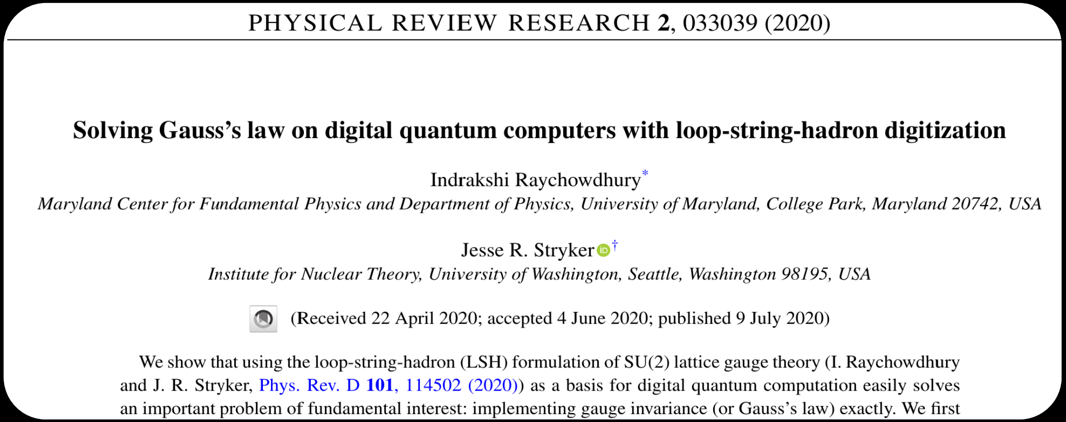
Hamiltonian in operator-factorized form is the input for developing simulation algorithms

<u>Advantages</u>

- All constraints are *Abelian*
  - Simultaneously diagonalizable
  - LSH basis states are individually definitely allowed or definitely unallowed, unlike other formulations
- Hilbert space is structure is far simpler than |jmm’> states
- Hamiltonian structure looks more similar to U(1)
- Clebsch-Gordons recast as SHO scaling factors
- First SU(2) physicality quantum circuits constructed (Raychowdhury & JS 2020)

SU(2) LSH & quantum computation

- Circuits for LSH constraints, in any number of dimensions, are worked out in detail
- Speedups likely needed to make possible in NISQ era


PHYSICAL REVIEW RESEARCH **2**, 033039 (2020)

Solving Gauss's law on digital quantum computers with loop-string-hadron digitization

Indrakshi Raychowdhury*
Maryland Center for Fundamental Physics and Department of Physics, University of Maryland, College Park, Maryland 20742, USA

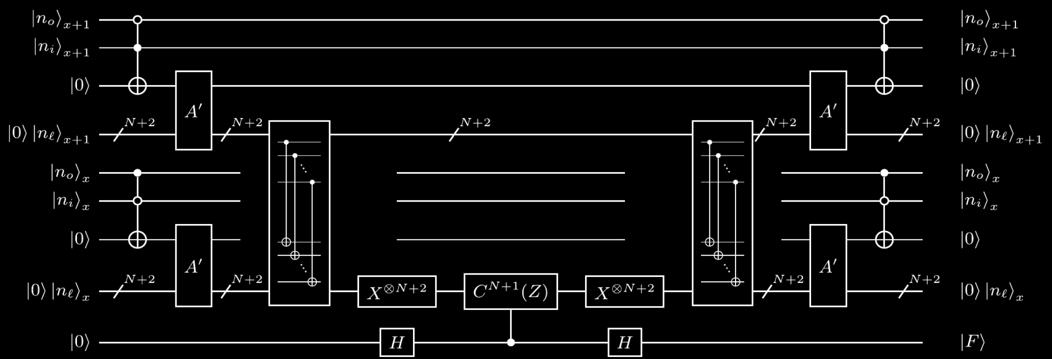
Jesse R. Stryker†
Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195, USA

(Received 22 April 2020; accepted 4 June 2020; published 9 July 2020)

We show that using the loop-string-hadron (LSH) formulation of SU(2) lattice gauge theory (I. Raychowdhury and J. R. Stryker, Phys. Rev. D **101**, 114502 (2020)) as a basis for digital quantum computation easily solves an important problem of fundamental interest: implementing gauge invariance (or Gauss's law) exactly. We first

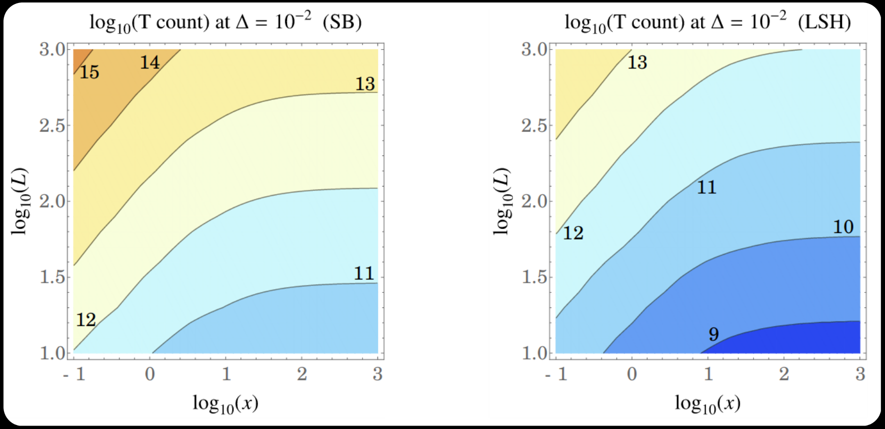
Potential LSH drawbacks:

- H_B in d>1 has **many** terms
- Can cost more qubits in d>1



1+1 SU(2): LSH vs Schwinger bosons

							Schwinger bosons		LSH	
x	η	L	t/a_s	Δ	$\alpha_{\text{Trot.}}$	$\alpha_{\text{Newt.}}$	Qubits	T gates	Qubits	T gates
1	4	100	1	0.01	90%	9%	2626	8.19713×10^{11}	1319	3.91817×10^{10}
1	4	100	1	0.001	90%	9%	2704	3.09951×10^{12}	1397	1.5172×10^{11}
1	4	100	10	0.01	90%	9%	2704	3.0993×10^{13}	1397	1.51643×10^{12}
1	4	100	10	0.001	90%	9%	2808	1.2146×10^{14}	1475	5.76229×10^{12}
1	4	1000	1	0.01	90%	9%	18904	3.12769×10^{13}	6797	1.53099×10^{12}
1	4	1000	1	0.001	90%	9%	19008	1.22564×10^{14}	6875	5.81562×10^{12}
1	4	1000	10	0.01	90%	9%	19008	1.22564×10^{15}	6875	5.81468×10^{13}
1	4	1000	10	0.001	90%	9%	19086	4.48657×10^{15}	6979	2.29217×10^{14}
1	8	100	1	0.01	90%	9%	4398	5.79224×10^{12}	1807	2.72735×10^{11}
1	8	100	1	0.001	90%	9%	4476	2.1482×10^{13}	1885	1.03709×10^{12}
1	8	100	10	0.01	90%	9%	4476	2.14816×10^{14}	1885	1.03705×10^{13}
1	8	100	10	0.001	90%	9%	4580	8.22615×10^{14}	1963	3.87886×10^{13}
1	8	1000	1	0.01	90%	9%	35076	2.16773×10^{14}	10885	1.04652×10^{13}
1	8	1000	1	0.001	90%	9%	35180	8.30098×10^{14}	10963	3.91414×10^{13}
1	8	1000	10	0.01	90%	9%	35180	8.30094×10^{15}	10963	3.91412×10^{14}
1	8	1000	10	0.001	90%	9%	35258	2.99214×10^{16}	11067	1.5154×10^{15}



T-gate costs at fixed m/g=1. Other simulation parameters not explicitly shown are η = 8, t/a_s = 1, $\alpha_{\text{Trot.}}$ = 90%, $\alpha_{\text{Newt.}}$ = 9%, and $\alpha_{\text{synth.}}$ = 1%.

Z. Davoudi, A.F. Shaw, & JS
arXiv:2212.14030

~20x T gate reduction with LSH

Digital simulations: SU(2)

PHYSICAL REVIEW D **101**, 074512 (2020)

SU(2) non-Abelian gauge field theory in one dimension on digital quantum computers

Natalie Klco,[*] Martin J. Savage,[†] and Jesse R. Stryker[‡]
Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195-1550, USA

Lattice Quantum Chromodynamics and Electrodynamics on a Universal Quantum Computer

Angus Kan[1] and Yunseong Nam[2, 3]

Article | Open Access | Published: 11 November 2021

SU(2) hadrons on a quantum computer via a variational approach

Yasar Y. Atas, Jinglei Zhang, Randy Lewis, Amin Jahanpour, Jan F. Haase & Christine A. Muschik

Nature Communications **12**, Article number: 6499 (2021) | Cite this article

Digital simulations: SU(3)

PHYSICAL REVIEW D **103**, 094501 (2021)

Editors' Suggestion

Trailhead for quantum simulation of SU(3) Yang-Mills lattice gauge theory in the local multiplet basis

Anthony Ciavarella[1,*] Natalie Klco[2,†] and Martin J. Savage[1,‡]

PHYSICAL REVIEW D **104**, 094514 (2021)

Quantum algorithms for transport coefficients in gauge theories

Thomas D. Cohen,[1,*] Henry Lamm,[2,†] Scott Lawrence,[3,‡] and Yukari Yamauchi[1,§]

(NuQS Collaboration)

Lattice Quantum Chromodynamics and Electrodynamics on a Universal Quantum Computer

Angus Kan[1] and Yunseong Nam[2,3]

PHYSICAL REVIEW D **105**, 074504 (2022)

Preparation of the SU(3) lattice Yang-Mills vacuum with variational quantum methods

Anthony N. Ciavarella[*] and Ivan A. Chernyshev[†]

Article | Open Access | Published: 11 November 2021

SU(2) hadrons on a quantum computer via a variational approach

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Nature Communications **12**, Article number: 6499 (2021) | Cite this article

PHYSICAL REVIEW D **107**, 054513 (2023)

Preparations for quantum simulations of quantum chromodynamics in 1 + 1 dimensions. II. Single-baryon β-decay in real time

Roland C. Farrell[1,*] Ivan A. Chernyshev[1,†] Sarah J. M. Powell[2,‡] Nikita A. Zemlevskiy[1,§] Marc Illa[1,‖] and Martin J. Savage[1,¶]

Summary: Progress toward QCD

arXiv ID	Author	Author	Author	Author	Title	U(1)	SU(2)	**SU(3)**	**NF**	1D	2D	**3D**	**evol.**	**prep.**	**meas.**	alg.	**QPU**
quant-ph/0510027	Byrnes	Yamamoto			Simulating Lattice Gauge Theories on a Quantum Computer	☑	☑	☑	0	☑	☑	☑	☑	☑	☐	◩	☐
1605.04570	Martinez	Muschik	Schindler	et al.	Real-Time Dynamics of Lattice Gauge Theories with a Few-Qubit Quantum Computer	☑	☐	☐	1	☑	☐	☐	☑	☐	☐	☐	☑
1803.03326	Klco	Dumitrescu	McCaskey	et al.	Quantum-Classical Computation of Schwinger Model Dynamics Using Quantum Computers	☑	☐	☐	1	☑	☐	☐	☑	☐	☐	☐	☑
1908.06935	Klco	Savage	**JRS**		SU(2) Non-Abelian Gauge Field Theory in One Dimension on Digital Quantum Computers	☐	☑	☐	0	☑	◩	☐	☑	☐	☐	☑	☑
2001.00698	Kharzeev	Kikuchi			Real-Time Chiral Dynamics from a Digital Quantum Simulation	☑	☐	☐	1	☑	☐	☐	☑	☐	☑	☑	☐
2002.11146	Shaw, AF	Lougovski	**JRS**	Wiebe	Quantum Algorithms for Simulating the Lattice Schwinger Model	☑	☐	☐	1	☑	☐	☐	☑	☐	☑	☑	☐
2005.10271	Mathis	Mazzola	Tavernelli		Toward Scalable Simulations of Lattice Gauge Theories on Quantum Computers	☑	☐	☐	1	☑	☑	☑	☑	☐	☐	☑	☐
2101.10227	Ciavarella	Klco	Savage		Trailhead for Quantum Simulation of SU(3) Yang-Mills Lattice Gauge Theory in the Local Mul	☐	☐	☑	0	☐	◩	☐	☑	☐	☐	☑	☑
2102.08920	Atas	Zhang, J	Lewis, R	et al.	SU(2) Hadrons on a Quantum Computer via a Variational Approach	☐	☑	☐	1	☐	☐	☐	☐	☑	☐	☑	☑
2104.02024	Cohen, T	Lamm	Lawrence	Yamauchi	Quantum algorithms for transport coefficients in gauge theories	☐	☑	☑	-	☐	☑	☑	☐	☐	☑	☑	☐
2107.12769	Kan	Nam, Y			Lattice Quantum Chromodynamics and Electrodynamics on a Universal Quantum Computer	☑	☑	☑	1	☑	☑	☑	☑	☐	☐	☑	☐
2110.06942	Tong, Y	Albert, V	McClean	et al.	Provably Accurate Simulation of Gauge Theories and Bosonic Systems	☑	☑	☐	0	☑	☑	☑	☑	☐	☐	☑	☐
2112.09083	Ciavarella	Chernyshev			Preparation of the SU(3) Lattice Yang-Mills Vacuum with Variational Quantum Methods	☐	☐	☑	0	☐	◩	☐	☐	☑	☐	☑	☑
2206.12454	Clemente	Crippa	Jansen		Strategies for the Determination of the Running Coupling of (2+1)-dimensional QED with Qua	☑	☐	☐	1	☐	☑	☐	☐	☐	☑	☑	☐
2207.01731	Farrell	Chernyshev	Powell, S	et al.	Preparations for Quantum Simulations of Quantum Chromodynamics in 1+1 Dimensions: (I)	☐	☐	☑	**2**	☑	☐	☐	☑	☑	☐	☑	☑
2207.03473	Atas	Haase	Zhang, J	et al.	Real-Time Evolution of SU(3) Hadrons on a Quantum Computer	☐	☐	☑	1	☑	☐	☐	☑	☐	☐	☑	☑
2209.10781	Farrell	Chernyshev	Powell, S	et al.	Preparations for Quantum Simulations of Quantum Chromodynamics in 1+1 Dimensions: (II)	☐	☐	☑	**2**	☑	☐	☐	☑	☑	☐	☑	☑
2211.10497	Kane	Grabowska	Nachman	Bauer	Efficient quantum implementation of 2+1 U(1) lattice gauge theories with Gauss law constrain	☑	☐	☐	0	☐	☑	☐	☑	☐	☑	☑	☑
2212.14030	Davoudi	Shaw, AF	**JRS**		General Quantum Algorithms for Hamiltonian Simulation with Applications to a Non-Abelian L	☐	☑	☐	1	☑	☐	☐	☑	☐	☐	☑	☐

A selection of papers that have advanced the field closer to DQS of lattice QCD. Checkmarks indicate applicability to a given feature. Green indicates key milestones; gold indicates end-goals of a complete lattice QCD simulation.
Notable omissions: scalar field theory, finite groups, formal developments, analog simulations.

evol. = time evolution, prep. = nontrivial state preparation, meas. = nontrivial observable measurement, alg. = constructive algorithms, QPU = includes hardware implementation.

Impressive progress, but scaling hardware beyond 1-2 sites and lowest cutoffs - not worked out yet

Takeaway messages

- Theory developments and algorithms are still in very early stages
- Many different interesting questions to address:
  Gauss's law, basis choice, truncations, simulation protocols
- These are vibrant research directions and we are learning more about gauge theories every day – even before quantum-advantage simulations

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