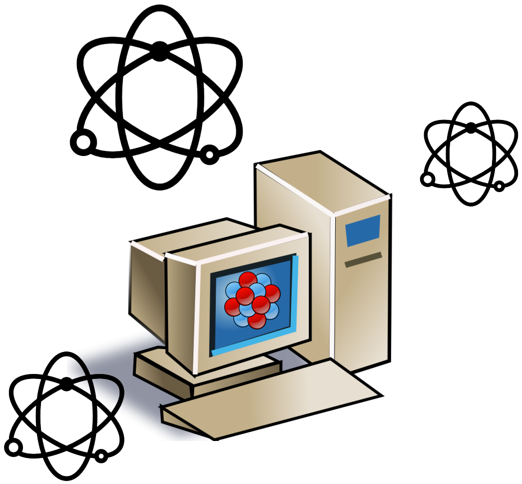
Compiling quantum gauge theories for quantum computation



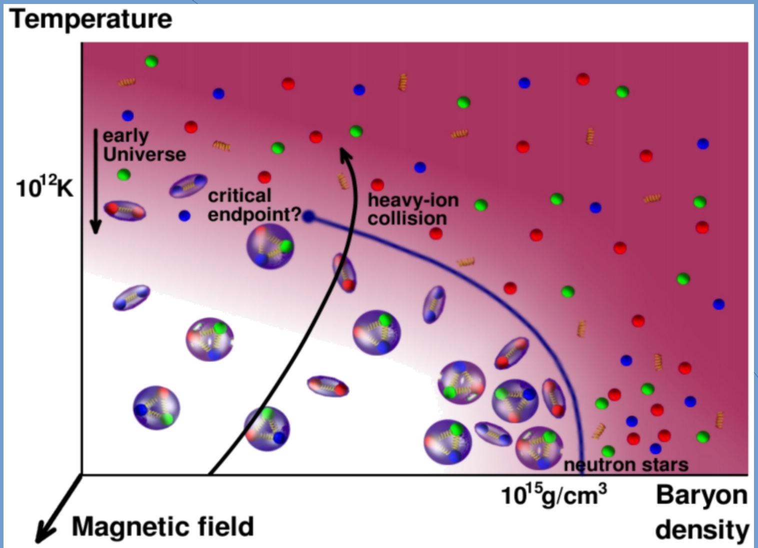
Jesse Stryker

University of Washington Final Exam
(advisor: David Kaplan)
2020/07/14

Big picture

Physics targets:

- **Simulation of quantum chromodynamics (QCD)**
  - Hadronization
  - Microscopic understanding of nuclear interactions
- **Complete phase diagram of QCD**
- **Nuclear equation of state**



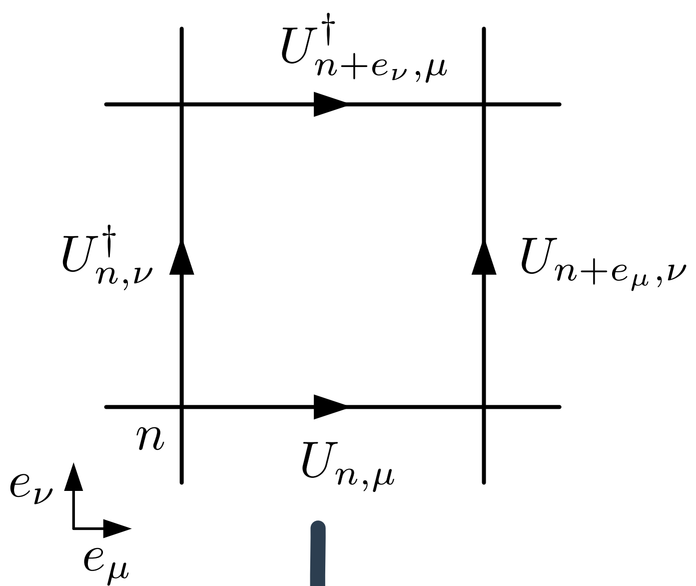
How to make these predictions?

- **Nonperturbative problems**
- **Numerically simulate QCD degrees of freedom**

Conjectured phase diagram credit: G. Endrödi J.Phys.Conf.Ser. 503 (2014) 012009

Traditional Lattice Field Theory

$x^\mu \to an^\mu$



- Defines a field theory nonperturbatively
- Spacetime is **discretized** with a lattice (e.g. square, cubic, hypercubic)
- Matter particles such as quarks are described by quantum fields that “live” on the **sites**
- Quantum gauge fields describing force-mediating bosons live on oriented **links** joining sites
- Gauge field’s numerical values, indicated by $U_{n,\mu}$ can be thought of as **matrices belonging to some Lie group**–the “gauge group” *G*

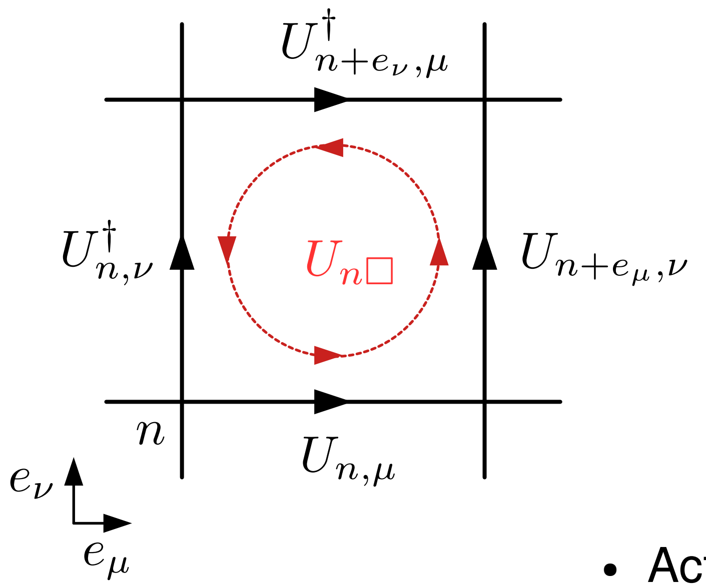
$$\begin{pmatrix} -0.7485 & -0.2744 - 0.6037i \\ 0.2744 - 0.6037i & -0.7485 \end{pmatrix}$$

A matrix from SU(2), the group of complex 2x2 matrices with determinant 1.

When *G* is non-Abelian, the gauge bosons self-interact

Traditional Lattice Field Theory

$x^\mu \to an^\mu$



Wilson's gauge action, S_W

"link operators" $U_{n,\mu}$ in gauge group G

$$S_W = -\beta \sum_{n,\mu} \mathrm{tr}(\underbrace{U_{n,\mu} U_{n+e_\mu,\nu} U^\dagger_{n+e_\nu,\mu} U^\dagger_{n,\nu}}_{U_\square \text{ "plaquette" operator}} + U^\dagger_\square)$$

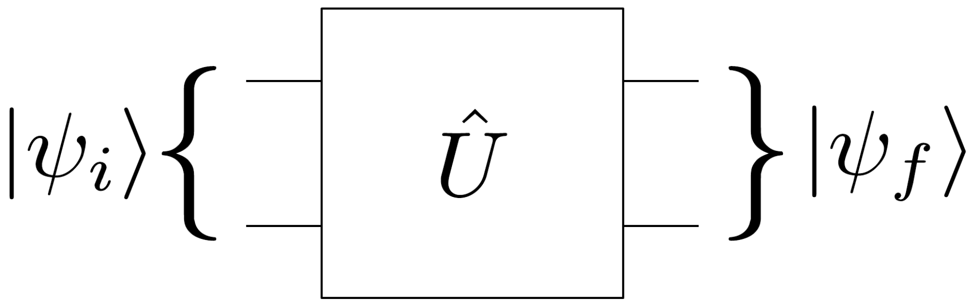
for non-Abelian

- Action: A function of the link operators' values, appearing in special combinations called **plaquettes**: products of link operators going around elementary (square) loops
- In classical simulations, exp(-S_W) acts like a probably weight for the configuration
- Real-time dynamics and nonzero baryon density both suffer from 'sign problems' in classical simulations, as explained at my *General Exam (Oct. 2017)*

INSTITUTE for NUCLEAR THEORY

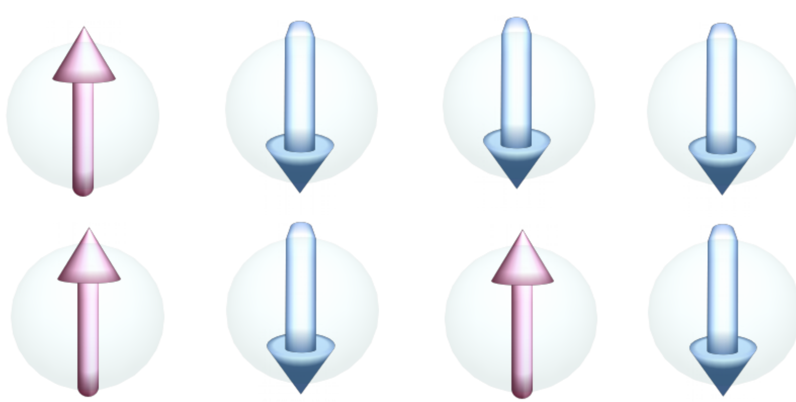
Classical problems.. quantum solutions?

Digital quantum computers:



Unitary gates: $e^{-it\hat{H}}$ with your favorite Hamiltonian

- Want to simulate nonperturbative gauge theory
  - Gauge theory on the lattice
  - Hamiltonian lattice gauge theory
- Has no apparent sign problems



General problem:

How to map a Hilbert space $\mathcal{H}$, and $\hat{H}$, on to qubits & quantum gates?

Hamiltonian lattice gauge theory

Lattice gauge theory Hilbert space structure

- An Abelian group, U(1)

Every link comes with its own Hilbert space.

$$\langle \phi | q \rangle = \frac{1}{\sqrt{2\pi}} e^{i\phi q}$$

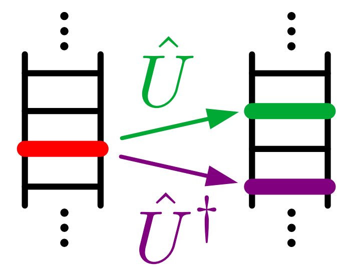
group element or “coordinate” basis for link

electric representation or “momentum” basis

$$[E, U] = U$$

$$U\,|q\rangle = |q+1\rangle$$

Quantized with canonical, same-link commutation relations.



“*U* raises *E*”

Gauge transformations:

$$\hat{U}_{n,i} \rightarrow e^{i(\theta_n - \theta_{n+e_i})}\hat{U}_{n,i}$$

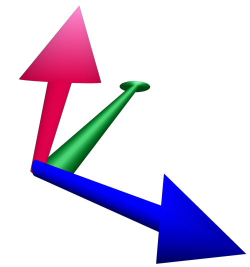
‘Kogut-Susskind’ Hamiltonian:

$$\hat{H}_E = \frac{g^2}{2}\sum_{n,i}\hat{E}^2_{n,i} \qquad \hat{H}_B = -\sum_n \frac{1}{2g^2}\mathrm{Re}(\hat{U}_{n,\square})$$

J. Kogut & L. Susskind (1975) Phys. Rev. D 11, 395

Hamiltonian lattice gauge theory

Lattice gauge theory Hilbert space structure
- Non-Abelian group such as SU(N)



Left and right electric fields each have 'colored' components in addition to spatial components

True gluons would have 8 such components

Gauge transformations: $\hat{U}_{n,i} \to \Omega_n \hat{U}_{n,i} \Omega^\dagger_{n+e_i}$

"Left" and "right" electric fields to generate the independent left/right rotations.

$$[E^a_{L/R}, E^b_{L/R}] = i f^{abc} E^c_{L/R}$$
$$[E^a_R, U] = U T^a$$
$$[E^a_L, U] = -T^a U$$

"*U* adds representations"

$$\langle g | j, m, n \rangle = \sqrt{\frac{d_j}{|G|}} D^{(j)}_{m,n}(g)$$

group element

representation state

$$\hat{H}_B = -\sum_n \frac{1}{2g^2} \mathrm{tr}(\hat{U}_{n,\Box} + \hat{U}^\dagger_{n,\Box})$$
$$\hat{H}_E = \frac{g^2}{2} \sum_{n,i} \hat{E}^a_{n,i} \hat{E}^a_{n,i}$$

More info: Zohar & Burrello, PRD 91, 054506 (2015)

Hamiltonian lattice gauge theory

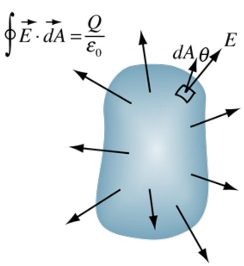
Plus Gauss law constraints

Gauss's law ↔ gauge invariance

charge conservation

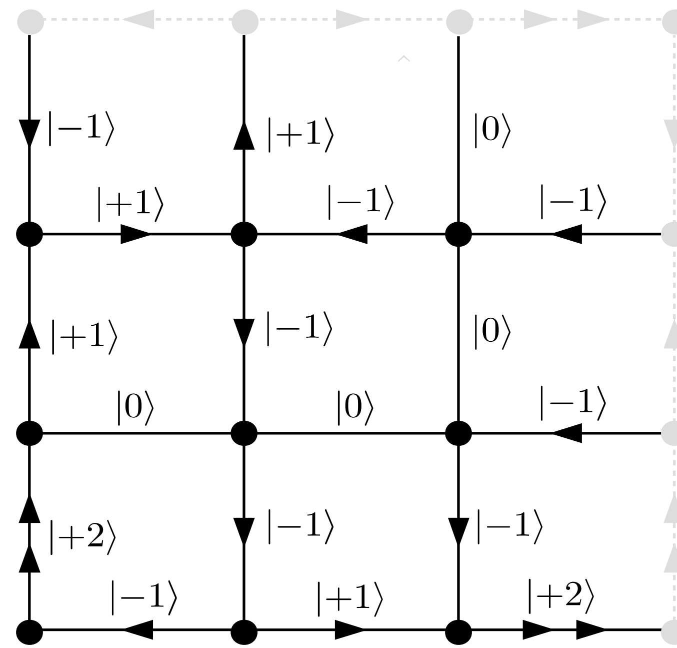
U(1)

$$\nabla \cdot \mathbf{E} - \rho = 0$$



$\hat{\mathcal{G}}_n$

$$\rho = \psi^\dagger \psi$$



compact U(1)
electric eigenbasis

SU(N)

$$\mathbf{D} \cdot \mathbf{E}^a - \rho^a = 0$$

$\hat{\mathcal{G}}_n^a$

$$\rho^a = \psi^\dagger T^a \psi$$

Issues with simulating a Kogut-Susskind framework

- **Qubits wasted on unphysical states**

- **Non-Abelian constraints mean individual basis states are virtually never allowed by themselves**
- **Quantum noise will create components along unphysical directions**
- **Gauge invariance not necessarily respected by algorithms, even for noiseless simulation**

Abelian, U(1): Implementing Gauss's law

JRS, Physical Review A 99, 042301 (2019)

Q: ***How to live with Gauss law constraints on a quantum computer?***

Immediate issues:

- **Preparing gauge invariant initial states**
- **Protecting digital quantum simulation from unphysical errors**
- **Mutilated time evolution possible**

These highlight a more basic problem.

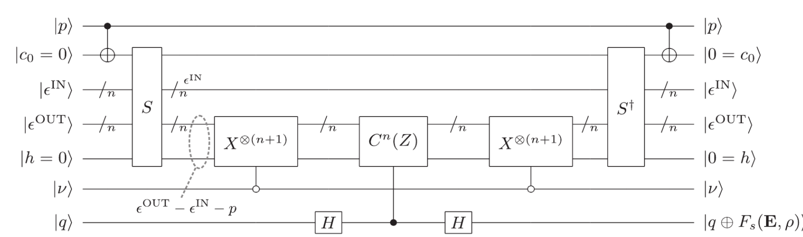
How to even *recognize* a legitimate state?

Abelian, U(1): Implementing Gauss's law

Cannot measure $\mathbf{E}$, ρ without collapsing a state $|\Psi\rangle$

→ Measure $\nabla\cdot\mathbf{E}$ - ρ with a Gauss law "oracle" JRS, PRA 99, 042301 (2019)

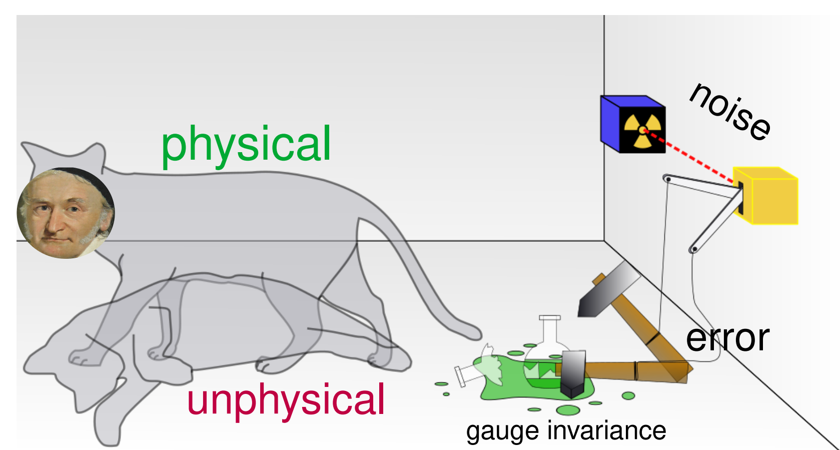
to project to $\mathcal{H}_{\text{physical}}$ or $\mathcal{H}_{\text{unphysical}}$



Internally 'flags' allowed states

|phys> → –|phys>

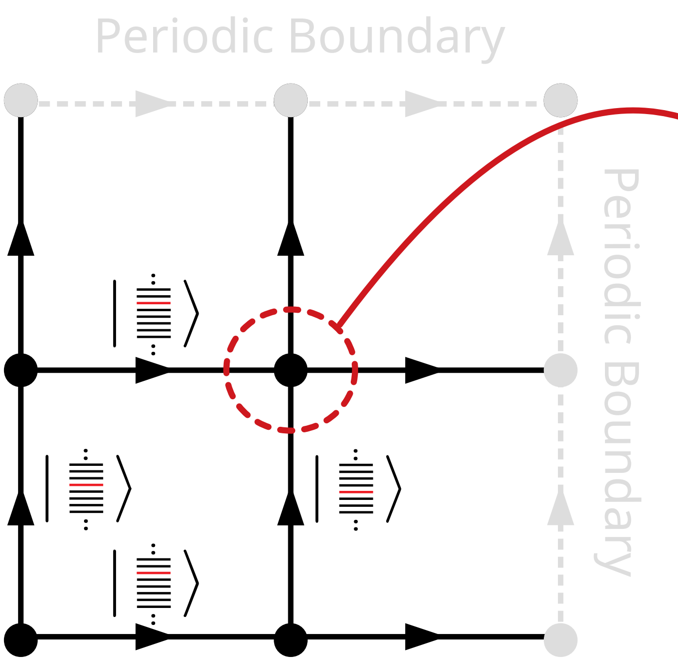
|unphys> → |unphys>



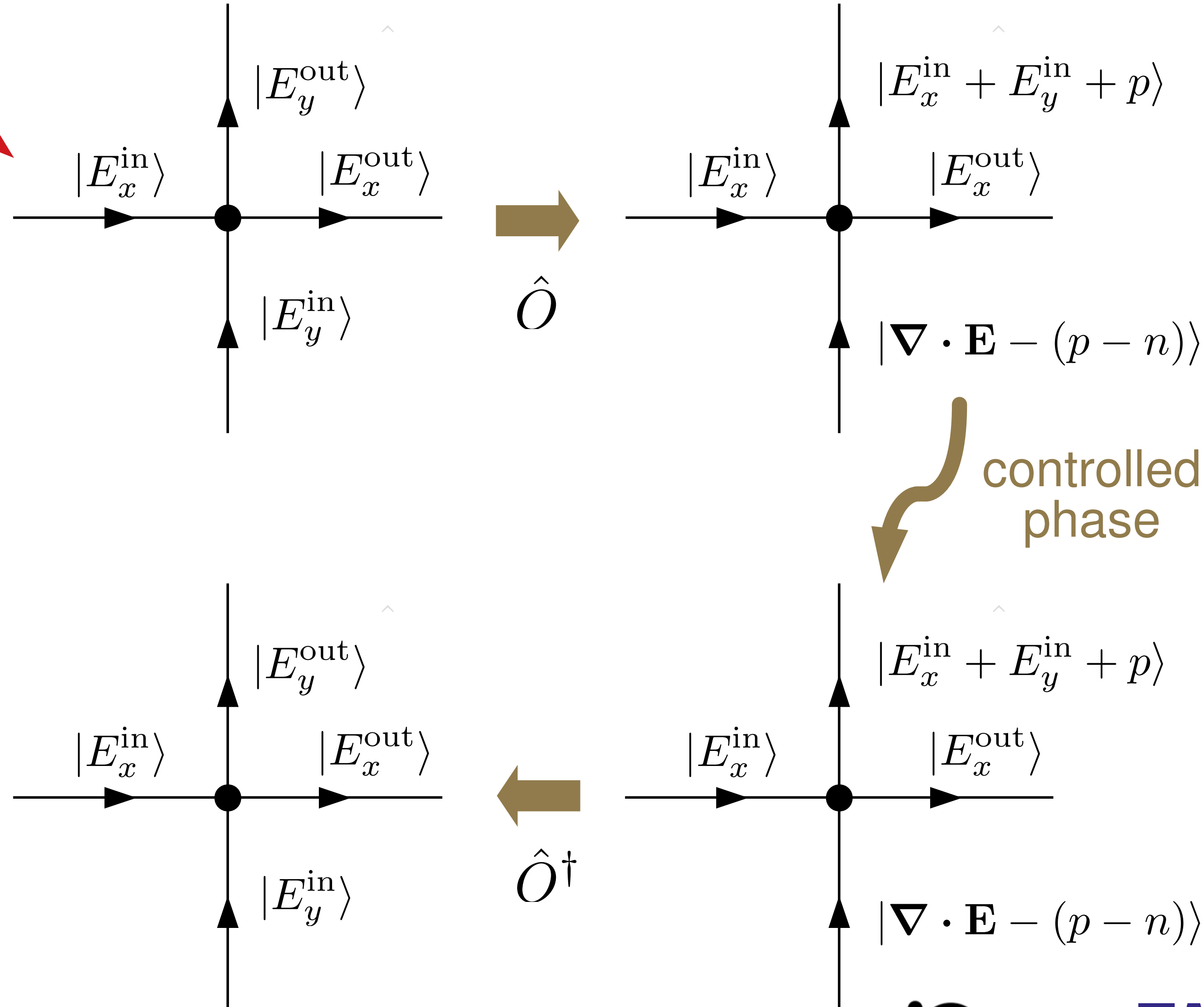
- **Discretizes continuous errors**
- **Can detect 'bit-flip' errors**

Abelian, U(1): Implementing Gauss's law

Basic procedure



$\hat{E}_\ell$ eigenbasis $(\in \mathbb{Z})$



Applications are still being explored

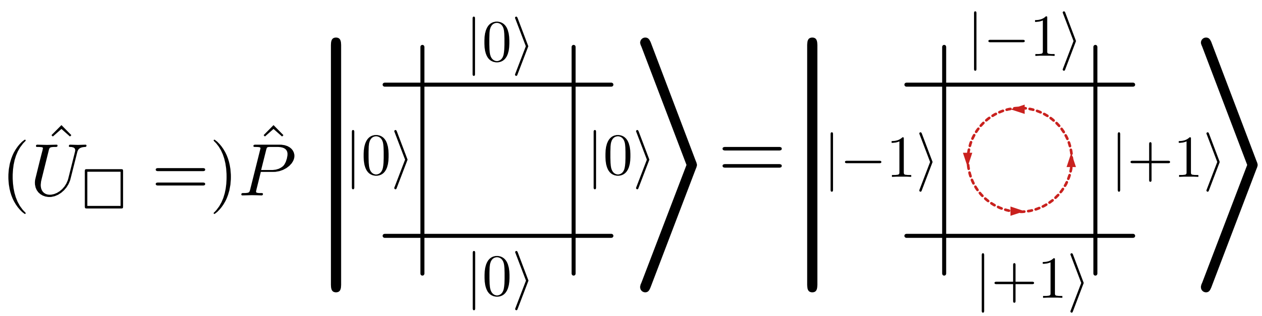
Abelian, U(1): Gauge-invariant variables

D.B. Kaplan & JRS
arXiv:1806.08797

<u>Alternate U(1) approach:</u> Ask, ‘what states does Hamiltonian *H* actually visit?’ A look at *H*:

- $\hat{H}_E \supset E^2_{n,i}$ scales electric states (trivial state mapping)

- $\hat{H}_B \supset P_n$ *excites electric flux loops*



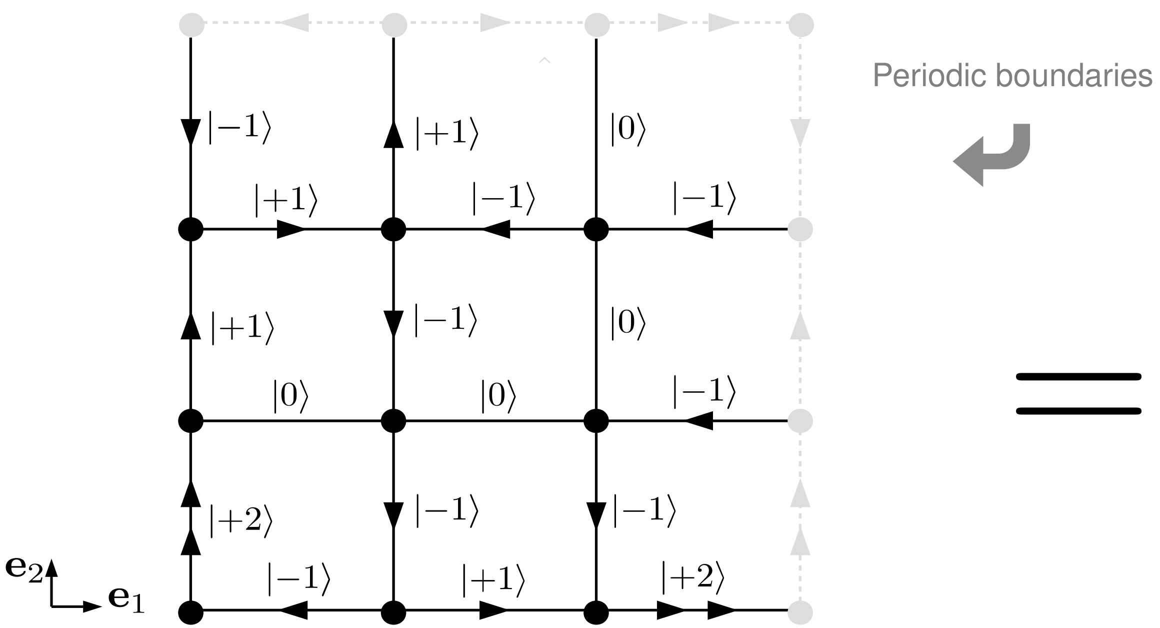
Flux loop excitations are oriented

Abelian, U(1): Gauge-invariant variables

Plaquette applications lead to quantized scalar field

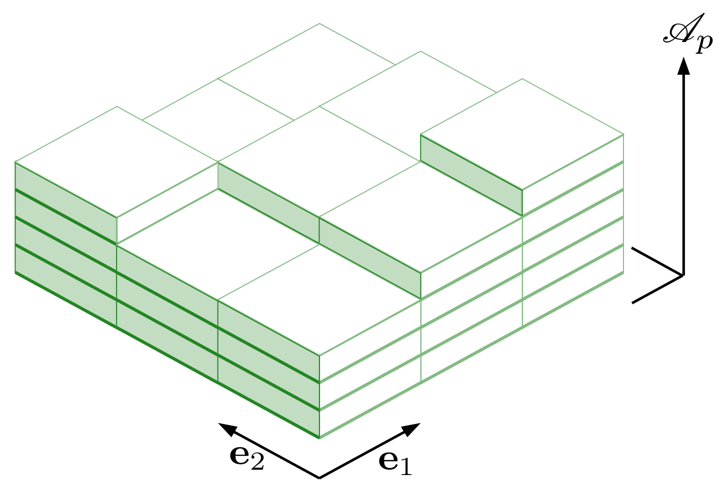
$$|\mathscr{A}_L\rangle \equiv \prod_{\boldsymbol{p}} \left(\hat{P}_{\boldsymbol{p}}\right)^{\mathscr{A}_{\boldsymbol{p}}} |\Omega\rangle$$

Plaquette powers, $\mathscr{A}_{\boldsymbol{p}}$: Encoding for valid $\mathcal{E}_{\boldsymbol{\ell}}$ configurations… or *now quantum numbers*



$\hat{E}_{\boldsymbol{\ell}}$ eigenbasis

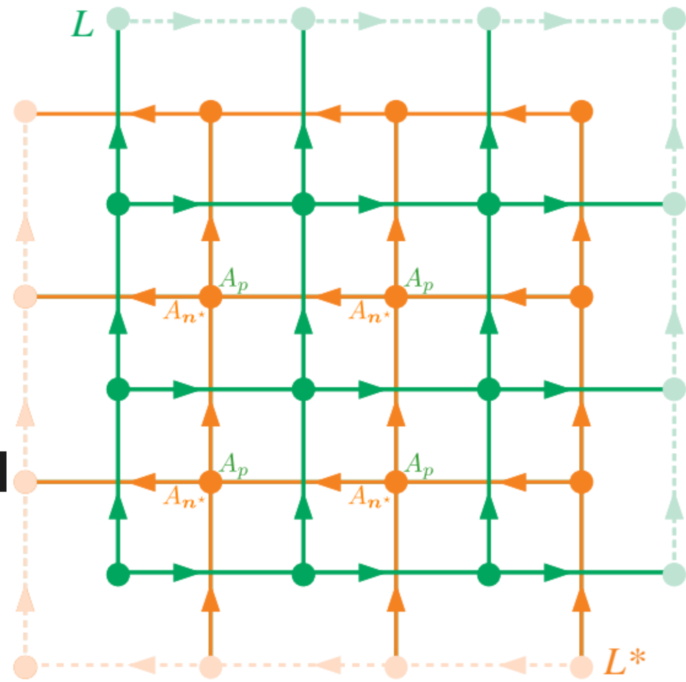
$=$



Number of plaquette operator applications

Abelian, U(1): Gauge-invariant variables

- **2D U(1) is dual to a scalar field theory (known) – dual uses only one integer per site**
  - Naturally emerges from gauge invariant building blocks
  - Resulting *H* describes the same physics more concisely



Limitations

- **2D with periodic boundary conditions (BC), and 3D, require magnetic Gauss laws**
  - **Enforcing for 2D (periodic BC) not practical with large volumes**
- **Unclear how generalize to include matter, non-Abelian groups**

Extensions being considered in new work, e.g. Haase, Dellantonio, et al. (2020) arXiv:2006.14160

Would want: Local Hilbert spaces, Hamiltonian built from local operators, local constraints (will come back later)

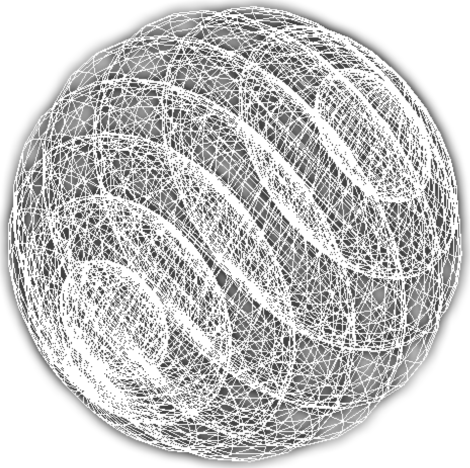
Non-Abelian, SU(2): Analytically-assisted quantum simulation

N. Klco, M.J. Savage & JRS
Physical Review D 101, 074512 (2020)

Some theoretical progress made in U(1). What about a non-Abelian group, SU(2)?
— We can start with Kogut-Susskind and see how far we get.

Q: *What can be simulated today?*

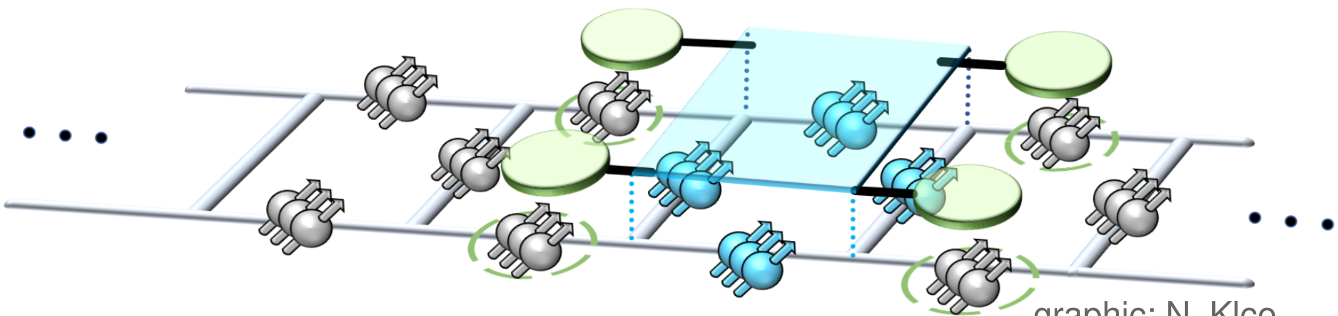
SU(2) is isomorphic to a three-sphere



INSTITUTE for NUCLEAR THEORY

Non-Abelian, SU(2): Analytically-assisted quantum simulation

System choice



graphic: N. Klco

We considered a periodic string of plaquettes or “ladder”

- It’s “only 1D,” but admits plaquettes
- 3-point vertices only – Gauss laws only involve three links
- Arbitrary length

Non-Abelian, SU(2): Analytically-assisted quantum simulation

Reductions

- Partial **solution to constraints**
  Analytic solution to Gauss's law at vertices reduces number of electric quantum numbers
  $$|j, m_L, m_R\rangle \to |j\rangle$$
  The *j*'s are still subject to triangle inequalities.

- Truncate electric flux *j* to one elementary unit per link.

- Number of plaquettes taken as the minimum L=2

Simulation targets

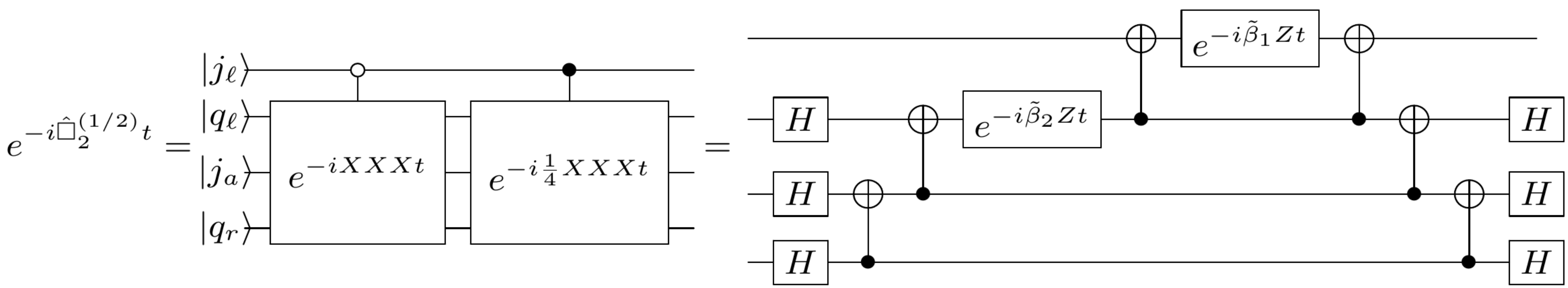
- How well does state stay in allowed space?
- Does electric energy evolution behave as expected?

- Simulate on IBM's 20-qubit Tokyo chip

Non-Abelian, SU(2): Analytically-assisted quantum simulation

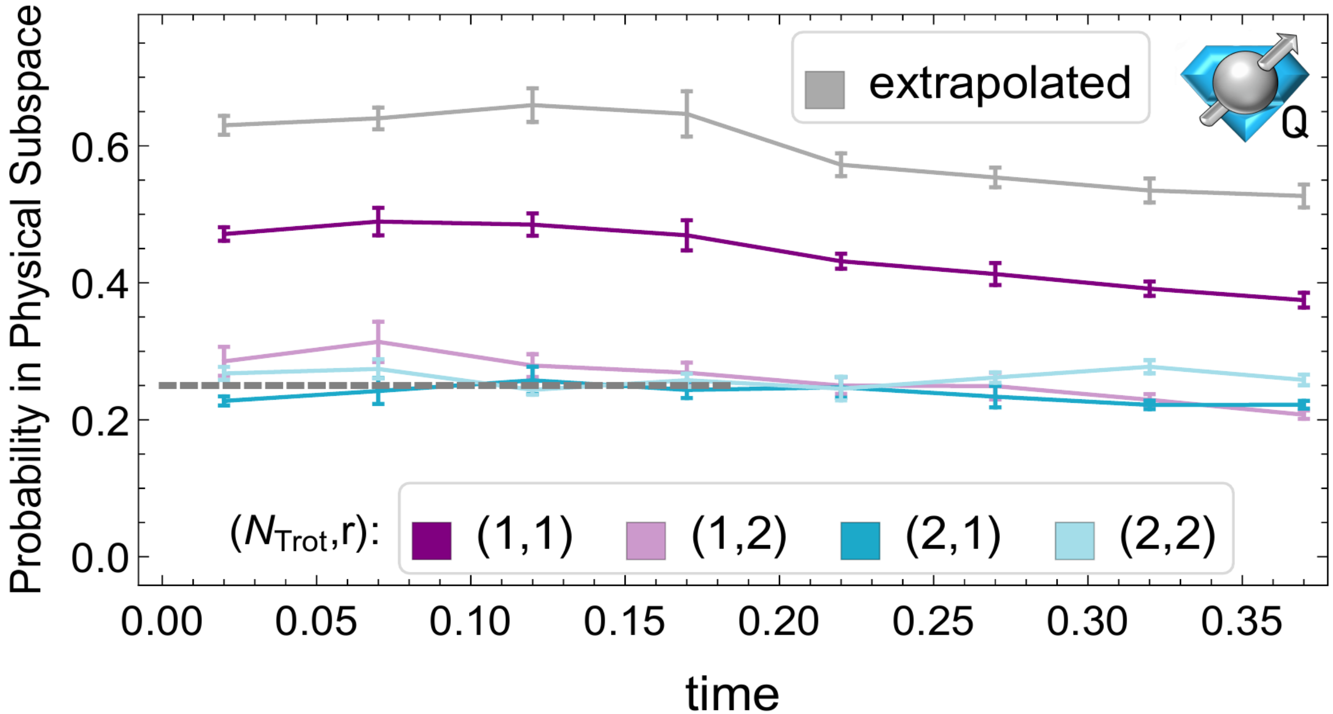
Example: Circuit for time evolution from one plaquette (Trotter-Suzuki time evolution)

$$\tilde{\beta}_2 = (5/8)t, \quad \tilde{\beta}_1 = (3/8)t,$$



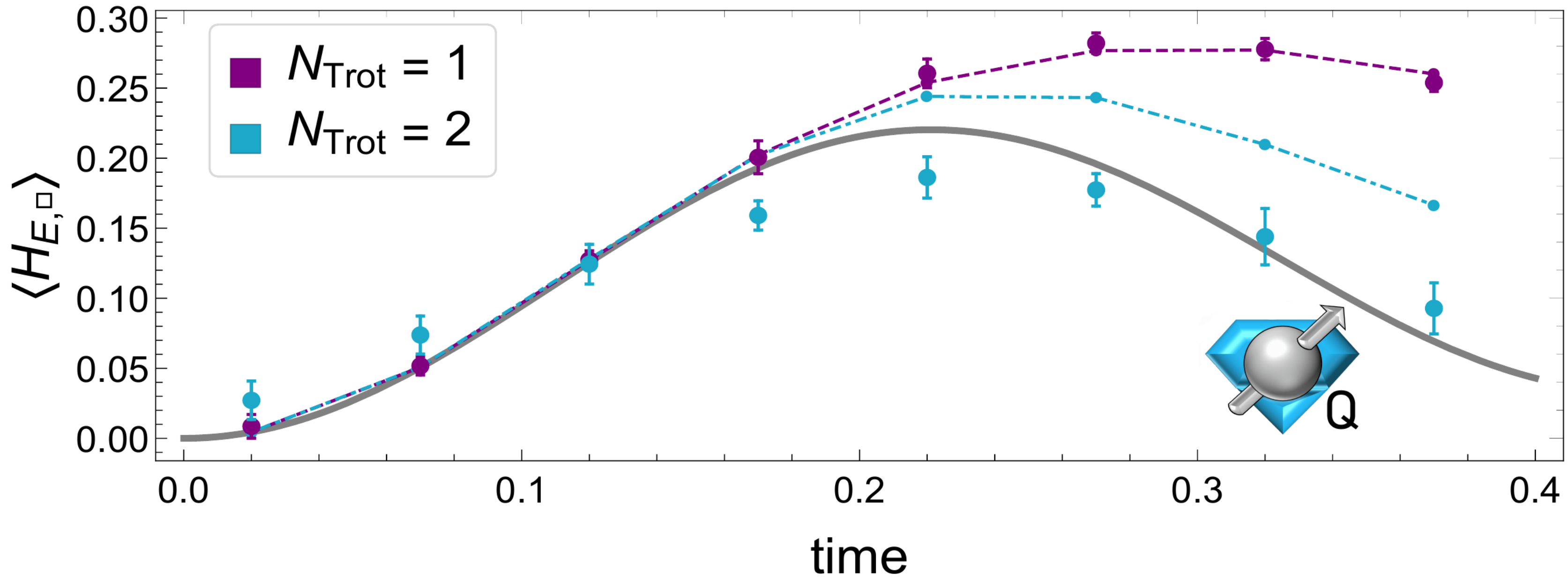
In this case:
- Circuit does not introduce further approximations
- Trotter-Suzuki decomposition respects gauge constraints

Non-Abelian, SU(2): Analytically-assisted quantum simulation



- Error mitigation techniques employed to reduce impact of noise
- Errors into unallowed space successfully mitigated for one ‘Trotter’ step of time evolution ($N_{\text{Trot}} = 1$)
- Results not reliable for more ($N_{\text{Trot}} > 1$)

Non-Abelian, SU(2): Analytically-assisted quantum simulation



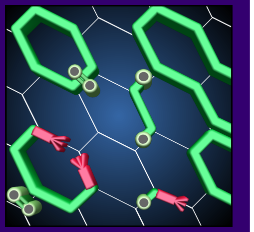
- We computed electric energy encircling one plaquette
  - using error-mitigated/extrapolated states
- Compared this to idealized (noiseless) simulation outcome
- $N_{\text{Trot}} = 1$ does get it right within uncertainties

Non-Abelian, SU(2): Analytically-assisted quantum simulation

Hardware results summary

- First simulation of a truncated SU(2) system, done on existing IBM hardware
- Used gauge theory constraints on Kogut-Susskind, and error mitigation, to make feasible and improve results
- Low enough circuit depth → Could extract an observable

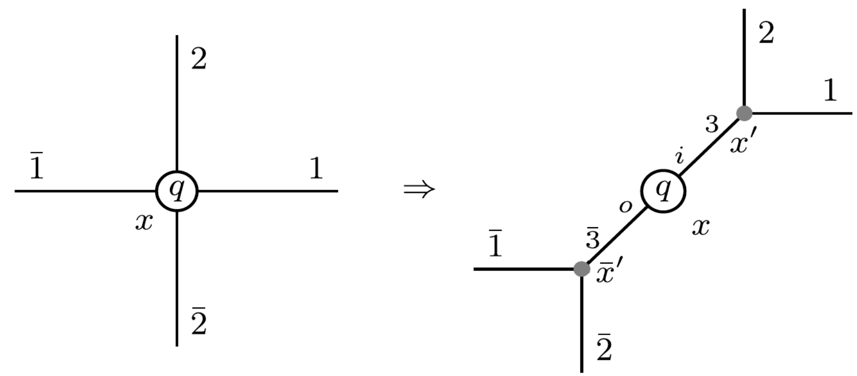
Non-Abelian, SU(2):
Loop-String-Hadron formulation



I. Raychowdhury & JRS
Physical Review D 101, 114502 (2020)
I. Raychowdhury & JRS
Physical Review Research 2, 033039 (2020)

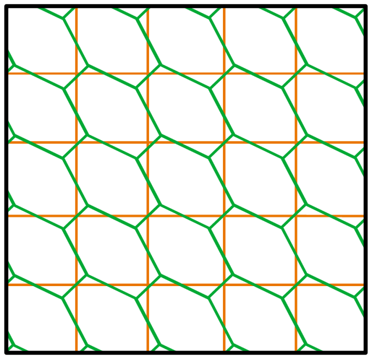
- *Loop formulations**:
  Reformulations of Kogut-Susskind-like Hamiltonian lattice gauge theory, in terms of fluctuating flux loop segments
  - SU(2) extensively studied

- *Loop-String-Hadron (LSH) formulation*:
  Our new generalization of SU(2) gauge theory to include "quarks," and valid in any number of dimensions



Basic rundown:

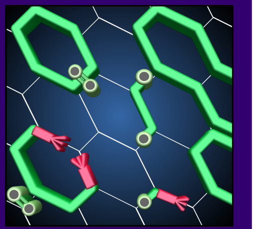
1) Start with square/cubic lattice
2) "Point split" vertices down to trivalent lattice
3) Three-point vertices are "gluonic" sites
4) Put "quark" sites at intermediate two-point vertices

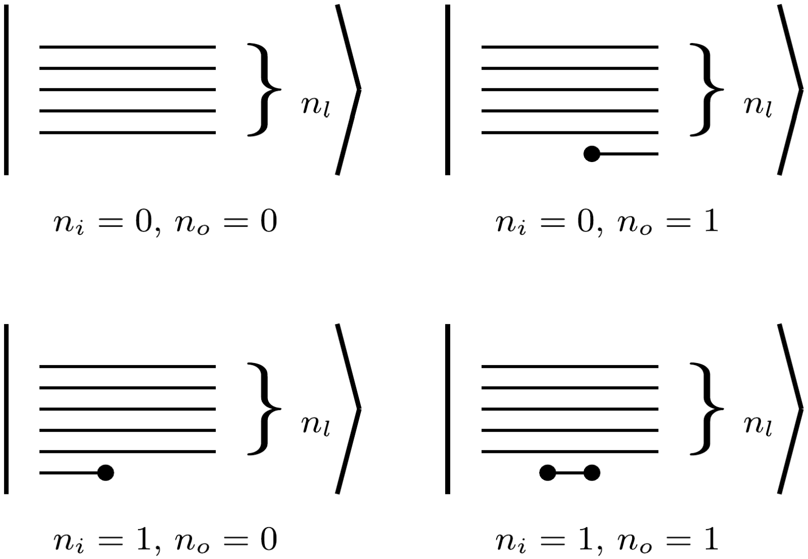


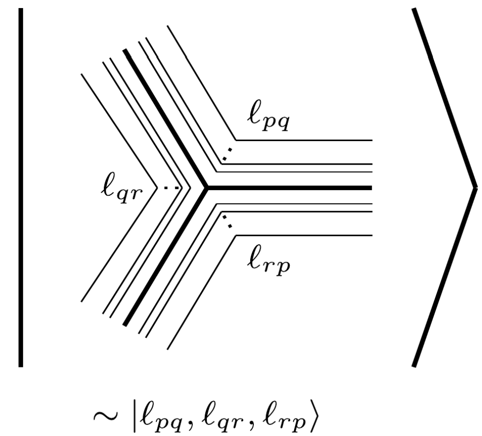
2D point splitting

* numerous papers by Anishetty, Mathur, Raychowdhury

Non-Abelian, SU(2): Loop-String-Hadron formulation







Quark sites have
- two quark degrees of freedom, n_i and n_o
- one loop degree of freedom, n_l,, the flux passing through the site

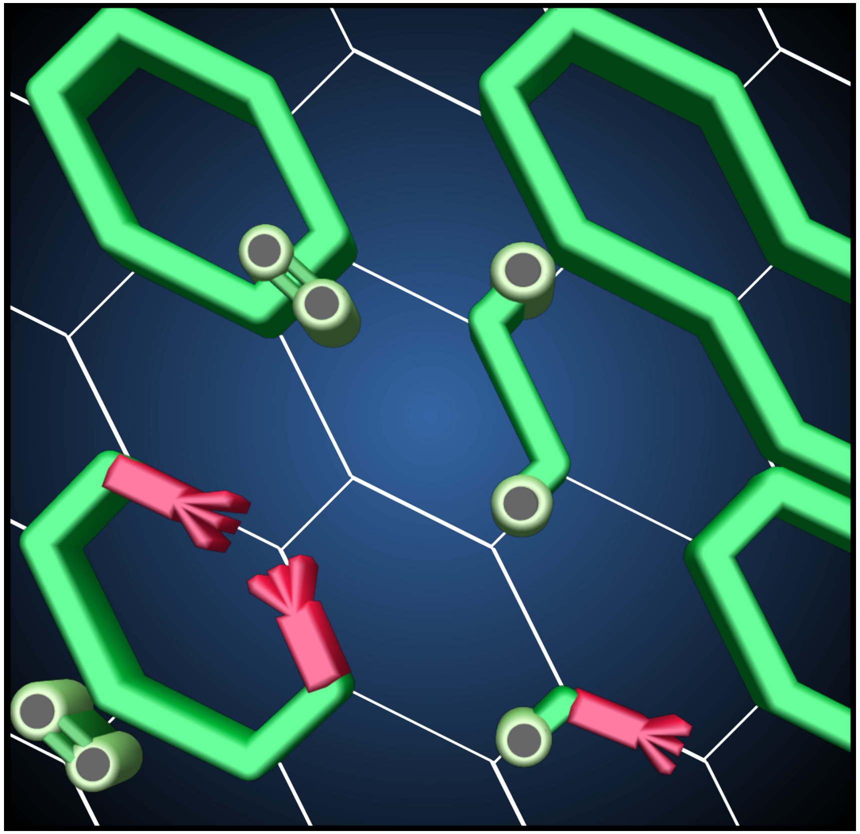
Gluonic sites have three loop degree of freedom, e.g. l_{12}, l_{23}, l_{31} that count flux units following particular paths

These site-local quantum numbers count excitations of pieces of gauge invariant objects

Constraints:
“Abelian Gauss law” requirements, flux should be conserved along links

Non-Abelian, SU(2): Loop-String-Hadron formulation

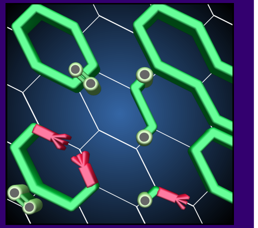
Constraints:
“Abelian Gauss law” requirements, flux needs to be conserved along links

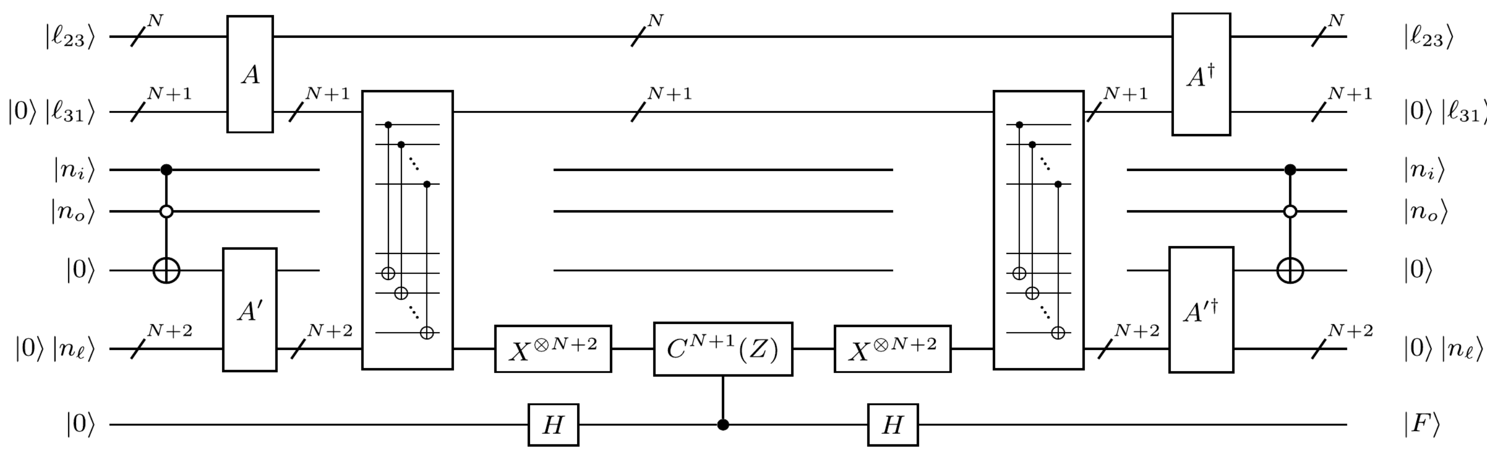


The Hilbert space has been cast into a form strongly resembling a truly Abelian [U(1)] gauge theory

Main difference: Electric flux has no ‘signed’ orientation

Non-Abelian, SU(2): Loop-String-Hadron formulation





- All constraints are made Abelian, so LSH basis states are each definitely allowed or definitely unallowed.

- Gauss law oracles just like those from U(1) carry over readily
  - First explicit and general quantum computer solution to constraints on non-Abelian gauge theory wave functions for quantum computers

- Hamiltonian also acquires some similarities with U(1); circuit translations for time evolution are now needed

Conclusions

- Rapid progress is being made in understanding the structure of gauge theories in the context of quantum simulation – both Abelian and non-Abelian
- Aspects of simulating conventional Kogut-Susskind formalism are being explored and analyzed
- Complementary approaches, such as Loop-String-Hadron, are also catching up and offer promising features for simulation
- Exactly gauge invariant wave functions can be implemented and verified (in theory), either for U(1) or SU(2)
- Early simulations of non-Abelian gauge theories have begun.

FIN

Thank you for your attention!

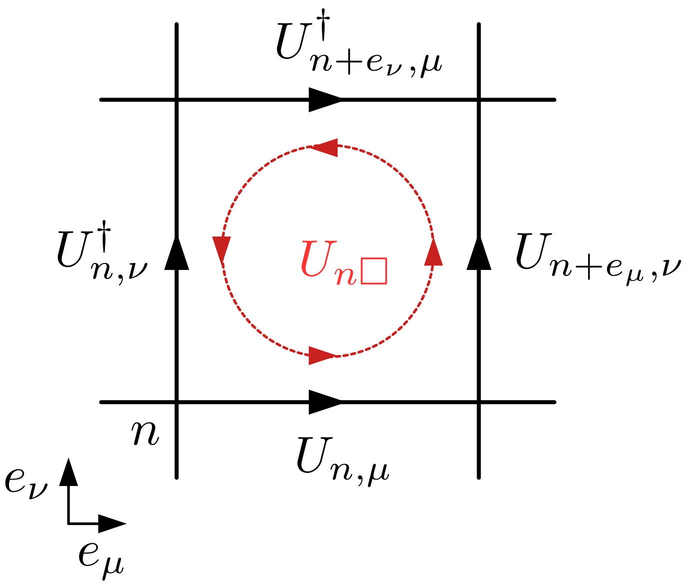
Questions?

Thank you to my thesis committee and to whoever made it to this slide. Special thanks are owed to the following people for helping me along my PhD journey: (Faculty) Silas Beane, David Kaplan, Gerald Miller, Ann Nelson, Martin Savage, Steve Sharpe; (Physics peers) Jon Craig, Nick Du, Julieta Gruszko, Ian Guinn, Dorota Grabowska, Natalie Klco, John Lombard, Kerkira Stockton, Michael Wagman, Michael Wilensky; (Collaborators) Pavel Lougovski, Indrakshi Raychowdhury, Alex Shaw, Nathan Wiebe; (Other) Lee Brown, Catherine Provost and the physics front office staff, my friends, and my family.

JRS has been supported by the Seattle Chapter of the Achievement Rewards for College Scientists (ARCS) Foundation; by the National Science Foundation Graduate Research Fellowship under Grant No.1256082 and Grant. No. DGE-1762114; and by the Institute for Nuclear Theory under DOE Grant No. DE-FG02-00ER41132.

Traditional lattice ingredients

$x^\mu \to a n^\mu$



$$Z = \int \prod_{n,\mu} [\mathcal{D}U_{n,\mu}] e^{-S[U]}$$

Monte Carlo on this

Wilson gauge action

"link operators" $U_{n,\mu}$ in gauge group G

$$S_W = -\beta \sum_{n,\mu} \mathrm{tr}(\underbrace{U_{n,\mu} U_{n+e_\mu,\nu} U^\dagger_{n+e_\nu,\mu} U^\dagger_{n,\nu}}_{U_\square \text{ "plaquette" operator}} + U^\dagger_\square)$$

for non-Abelian

Relation to continuum gauge fields:

$$U_{n,\mu} = e^{iaA_\mu(x)} \qquad S_W = a^4 \sum_n F^\alpha_{\mu\nu} F^{\mu\nu}_\alpha + \cdots$$

Fermionic matter:

- Grassmann integrals done analytically → "Fermion determinant"

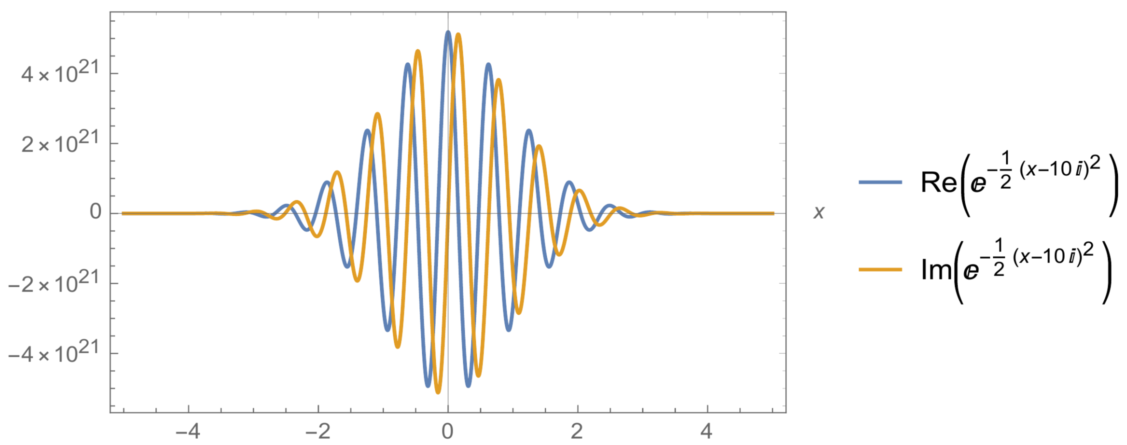
Traditional lattice

Euclidean path integral Monte Carlo

- **Great for static, equilibrium properties**
- **Real-time dynamics? Nonzero density? Topological term?**

-$S[U]$ generically complex-valued
→ "Sign problems"

Oscillations of a Gaussian: $S=\frac{1}{2}(x-10i)^2$

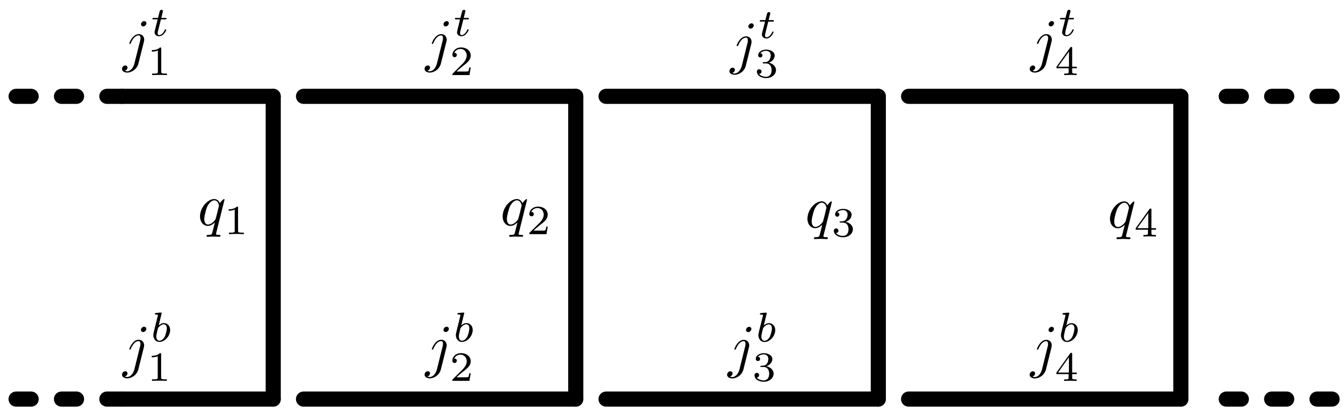


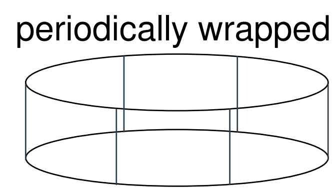
$$\int dx\, e^{-\frac{1}{2}(x-10i)^2} = \sqrt{2\pi} \ll 10^{21}$$

$$\langle e^{i\theta} \rangle = e^{-50}$$

Exponentially hard to sample oscillating path integral

Gauge singlet basis





Fully gauge invariant state of lattice with definite link angular momenta:

$$|\chi\rangle = \mathcal{N} \sum_{\{m\}} \prod_{i=1}^{L} \langle j_i^t, m_{i,R}^t, j_{i+1}^t, m_{i+1,L}^t | q_i, m_{q_i}^t \rangle$$

} CG's to form singlets at "top" vertices

$$\langle j_i^b, m_{i,R}^b, j_{i+1}^b, m_{i+1,L}^b | q_i, m_{q_i}^b \rangle$$

} CG's to form singlets at "bottom" vertices

$$|j_i^t, m_{i,L}^t, m_{i,R}^t\rangle \otimes |j_i^b, m_{i,L}^b, m_{i,R}^b\rangle \otimes |q_i, m_{q_i}^t, m_{q_i}^b\rangle$$

} Kets going around each "staple"

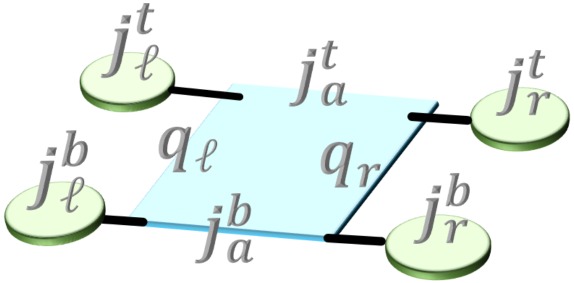
- Just using angular momentum addition (Clebsch-Gordan coefficients) to form singlets

Matrix elements of *H*

- **Non-diagonal elements derive from link operators in H_B : ***

$$\hat{U}_{\alpha\beta}|j,a,b\rangle = \sum_{\oplus J} \sqrt{\frac{\dim(j)}{\dim(J)}}\ |J, a+\alpha, b+\beta\rangle$$

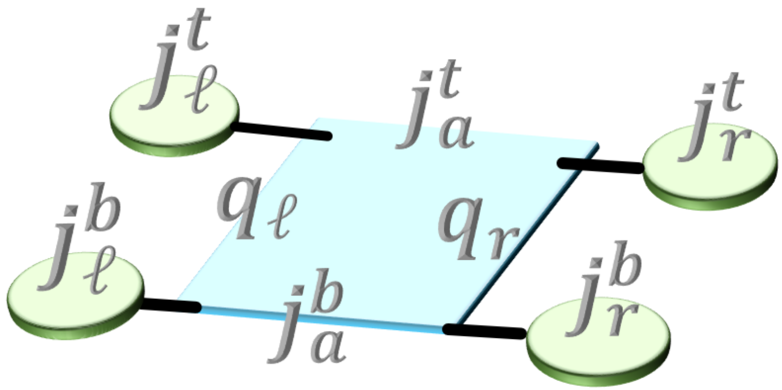
$$\times\ \ \langle j, a, \frac{1}{2}, \alpha | J, a+\alpha\rangle\langle j, b, \frac{1}{2}, \beta | J, b+\beta\rangle$$



each link op going round a plaquette "adds" ½-unit of angular momentum:
$J = j \pm 1/2$

- **This is all the info needed to compute matrix $||H||$ w.r.t. singlet states**

Plaquette operator & gauge-variant completion



- With singlet basis, matrix elements of $\hat{\square}$ depend on plaquette's j's, as well as adjacent j's
- Still have disallowed states
  - Action of plaquette op on disallowed space is arbitrary

    → "Gauge-variant completion" (GVC): Only worry about getting correct matrix elements between allowed states

Trotter-Suzuki time evolution

Time evolution operator replaced by Trotter-Suzuki approximation

$$e^{-i\ \Delta t (H_E + H_B)} \simeq e^{-i\ \Delta t H_E} e^{-i\ \Delta t \Box_1/(2g^2)} e^{-i\ \Delta t \Box_2/(2g^2)}$$

Try: *t* spread over one Trotter step, two Trotter steps, …
starting from strong-coupling vacuum (all j=0)

Abelian oracle decomposition, 2D U(1)

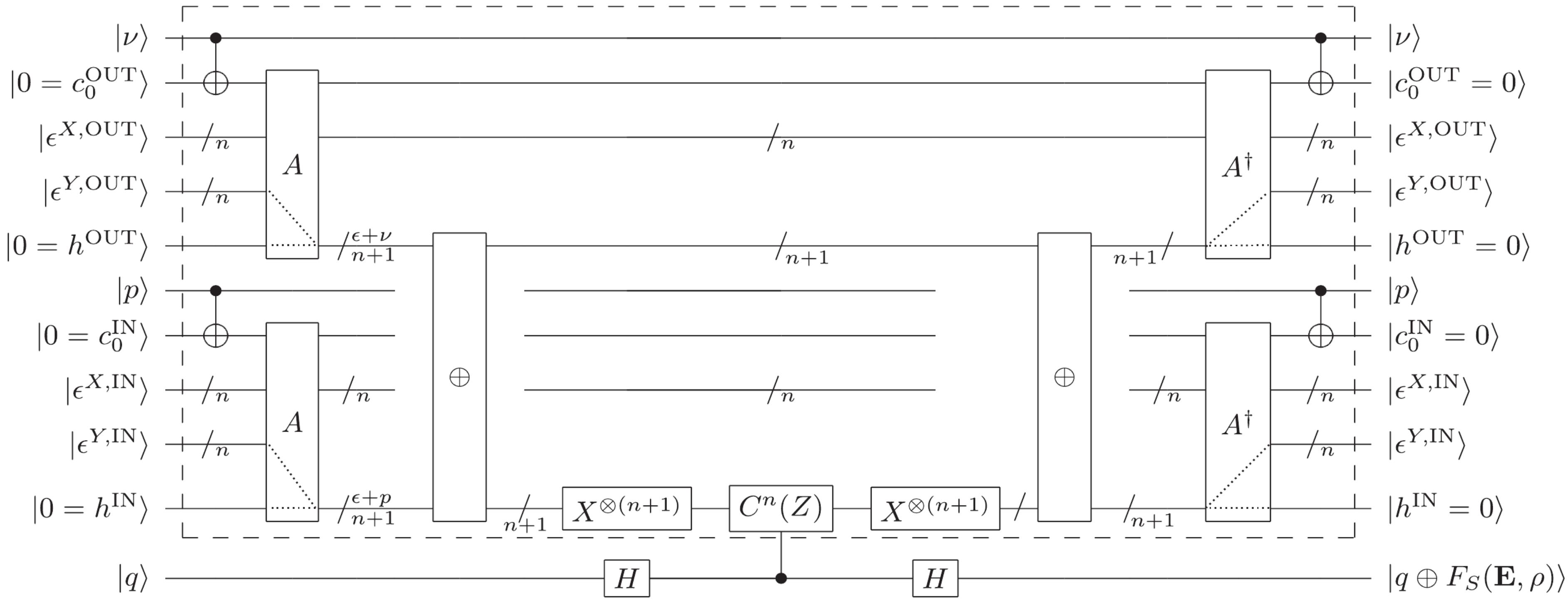
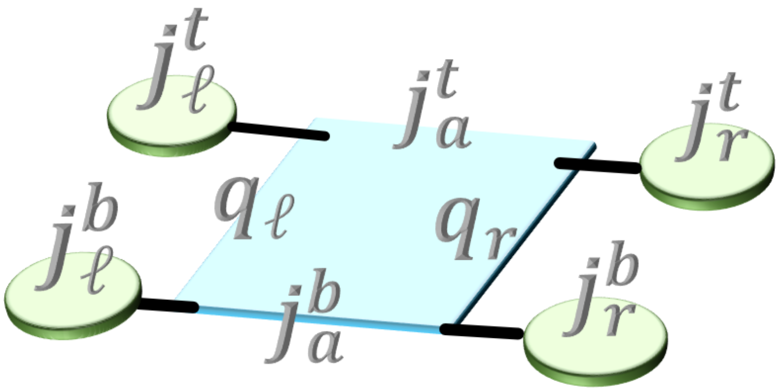


FIG. 5. A query to the oracle a for U(1) gauge theory in 2D. This example accommodates one flavor of Dirac fermion via the occupation numbers ν and p. To modify this for $g = \mathbb{Z}_{2^n}$, the overflow bits $h^{\mathrm{OUT,IN}}$ would be removed.

Specific truncation

$\Lambda_j = ½$



$\langle j_{\ell f}\ q_{\ell f}\ j_{af}\ q_{rf}\ j_{rf}\lvert\hat{\Box}^{(1/2)}\rvert j_{\ell i}\ q_{\ell i}\ j_{ai}\ q_{ri}\ j_{ri}\rangle$	
$\langle 00000\lvert\hat{\Box}^{(1/2)}\rvert 0\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}0\rangle,\ \langle 0\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}0\lvert\hat{\Box}^{(1/2)}\rvert 00000\rangle$	1
$\langle 000\tfrac{1}{2}\tfrac{1}{2}\lvert\hat{\Box}^{(1/2)}\rvert 0\tfrac{1}{2}\tfrac{1}{2}0\tfrac{1}{2}\rangle,\ \langle 0\tfrac{1}{2}\tfrac{1}{2}0\tfrac{1}{2}\lvert\hat{\Box}^{(1/2)}\rvert 000\tfrac{1}{2}\tfrac{1}{2}\rangle$	$\frac{1}{2}$
$\langle\tfrac{1}{2}\tfrac{1}{2}000\lvert\hat{\Box}^{(1/2)}\rvert\tfrac{1}{2}0\tfrac{1}{2}\tfrac{1}{2}0\rangle,\ \langle\tfrac{1}{2}0\tfrac{1}{2}\tfrac{1}{2}0\lvert\hat{\Box}^{(1/2)}\rvert\tfrac{1}{2}\tfrac{1}{2}000\rangle$	$\frac{1}{2}$
$\langle\tfrac{1}{2}0\tfrac{1}{2}0\tfrac{1}{2}\lvert\hat{\Box}^{(1/2)}\rvert\tfrac{1}{2}\tfrac{1}{2}0\tfrac{1}{2}\tfrac{1}{2}\rangle,\ \langle\tfrac{1}{2}\tfrac{1}{2}0\tfrac{1}{2}\tfrac{1}{2}\lvert\hat{\Box}^{(1/2)}\rvert\tfrac{1}{2}0\tfrac{1}{2}0\tfrac{1}{2}\rangle$	$\frac{1}{4}$

Relevant, nonzero matrix elements of the $\Lambda_j = 1/2$ truncated plaquette operator with $j_\ell^t = j_\ell^b \equiv j_\ell$

$$\hat{\Box}^{(1/2)} = \Pi_0 XXX\Pi_0 + \frac{1}{2}\Pi_0 XXX\Pi_1 + \frac{1}{2}\Pi_1 XXX\Pi_0 + \frac{1}{4}\Pi_1 XXX\Pi_1$$

$$|j=0\rangle \to |0\rangle\,, \quad |j=1/2\rangle \to |1\rangle$$

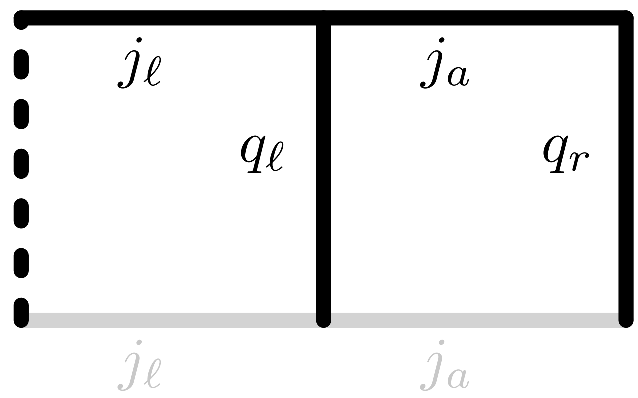
Specific truncation and volume

For our simulation:

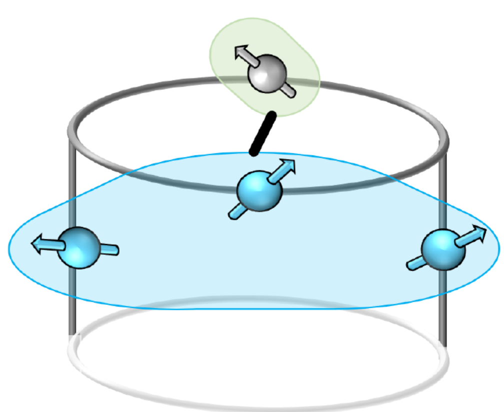
- Cutoff Λ_j = ½
- Length L=2

\+ Simplifications
= Four ‘active’ links

$|j_l\ q_l\ j_a\ q_r\rangle$



Four qubits represent state



$$\hat{\Box}^{(1/2)} = \Pi_0 XXX\Pi_0 + \cancel{\frac{1}{2}\Pi_0 XXX\Pi_1} + \cancel{\frac{1}{2}\Pi_1 XXX\Pi_0} + \frac{1}{4}\Pi_1 XXX\Pi_1$$

→GVC of plaquette operator:

$$\hat{\Box}^{(1/2)} = \Pi_0 XXX + \frac{1}{4}\Pi_1 XXX$$

Plaquette operator matrix elements

$$\langle \chi_{\dots,j_\ell^{t,b},q_{\ell f},j_{af}^{t,b},q_{rf},j_r^{t,b},\dots} | \hat{\Box} | \chi_{\dots,j_\ell^{t,b},q_{\ell i},j_{ai}^{t,b},q_{ri},j_r^{t,b},\dots} \rangle =$$

$$\sqrt{\dim(j_{ai}^t)\dim(j_{af}^t)\dim(j_{ai}^b)\dim(j_{af}^b)}$$

$$\times\sqrt{\dim(q_{\ell i})\dim(q_{\ell f})\dim(q_{ri})\dim(q_{rf})}$$

$$\times(-1)^{j_\ell^t+j_\ell^b+j_r^t+j_r^b+2(j_{af}^t+j_{af}^b-q_{\ell i}-q_{ri})}$$

$$\times 0.9 \begin{Bmatrix} j_\ell^t & j_{ai}^t & q_{\ell i} \\ \frac{1}{2} & q_{\ell f} & j_{af}^t \end{Bmatrix} \begin{Bmatrix} j_\ell^b & j_{ai}^b & q_{\ell i} \\ \frac{1}{2} & q_{\ell f} & j_{af}^b \end{Bmatrix} \begin{Bmatrix} j_r^t & j_{ai}^t & q_{ri} \\ \frac{1}{2} & q_{rf} & j_{af}^t \end{Bmatrix} \begin{Bmatrix} j_r^b & j_{ai}^b & q_{ri} \\ \frac{1}{2} & q_{rf} & j_{af}^b \end{Bmatrix}$$

Data processing

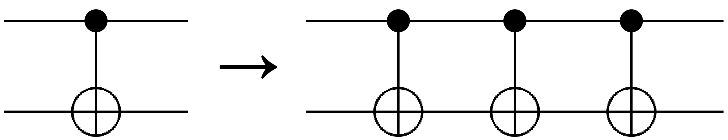
From IBM: Probabilities measured in computational basis

1) Constrained inversion → pre-measurement probabilities

- Needed because of measurement errors

2) Run simulation with superfluous CNOT pairs inserted

- $(\mathrm{CNOT})^2 = 1$, but introduces extra noise



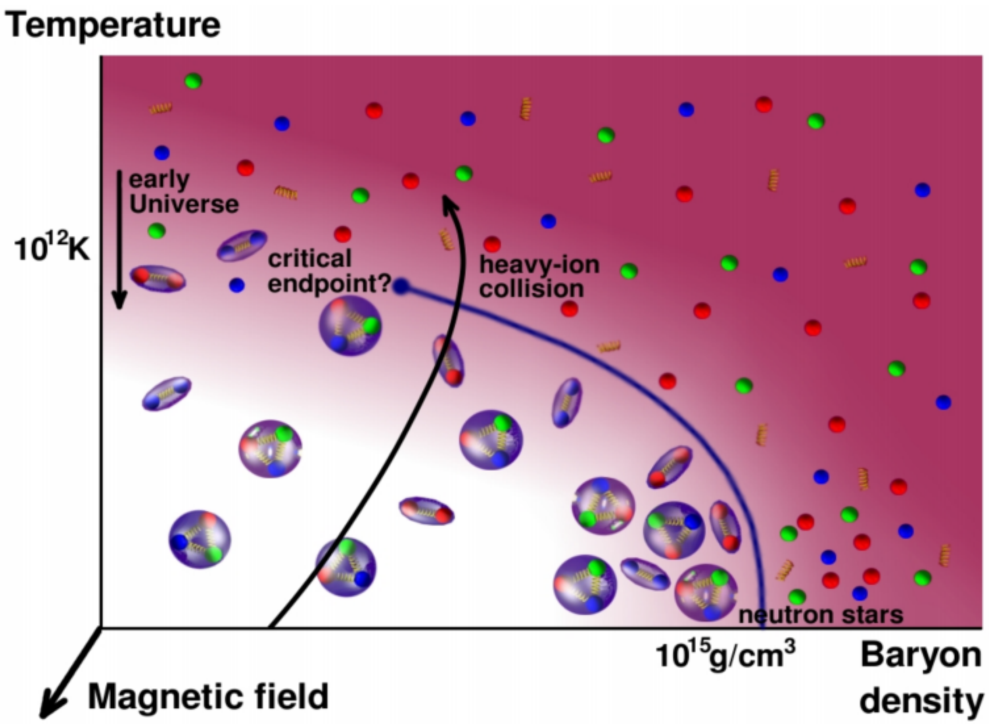
3) Extrapolate pre-measurement probabilities to zero CNOT noise

IBM Tokyo Q20 specs

	Qubit Connectivity			T1 (μsec)			T2 (μsec)		
Qubit Count	Min	Max	Ave	Min	Max	Ave	Min	Max	Ave
20	2	6	3.9	42.2	148.5	84.3	24.3	78.4	49.6

1-Qubit Gate Fidelity			2-Qubit Gate Fidelity			Readout Fidelity		
Min	Max	Ave	Min	Max	Ave	Min	Max	Ave
99.39%	99.94%	99.80%	92.88%	98.53%	97.16%	N/A	N/A	91.72%

Graphics credits



QCD phase diagram: Overview of recent lattice results - Scientific Figure on ResearchGate. Available from: https://www.researchgate.net/figure/Conjectured-QCD-phase-diagram_fig1_261701898 [accessed 23 Jan, 2019]