Quantum algorithms for Hamiltonian simulation of non-Abelian interactions

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UCLA virtual seminar
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arXiv:2212.14030
[to appear in Quantum]

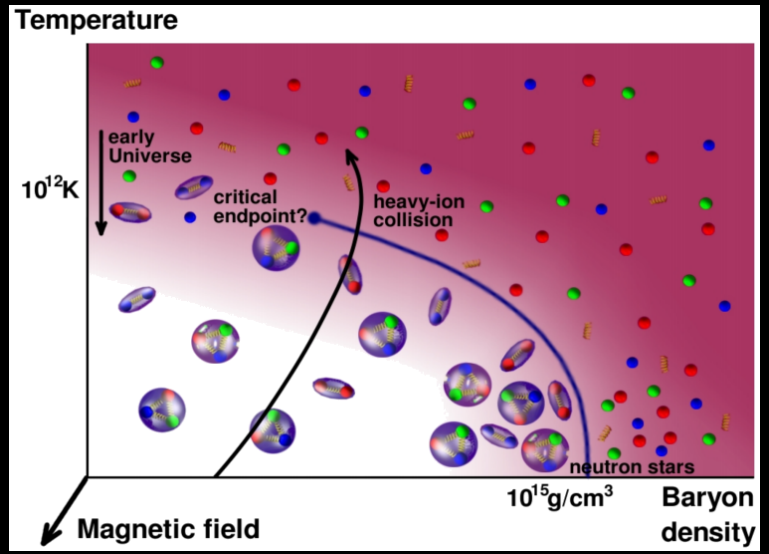
Big picture

Physics targets:

- Simulation of quantum chromodynamics
  - Hadronization
  - Microscopic understanding of nuclear interactions
- Complete phase diagram of QCD
- Equation of state for nuclear matter

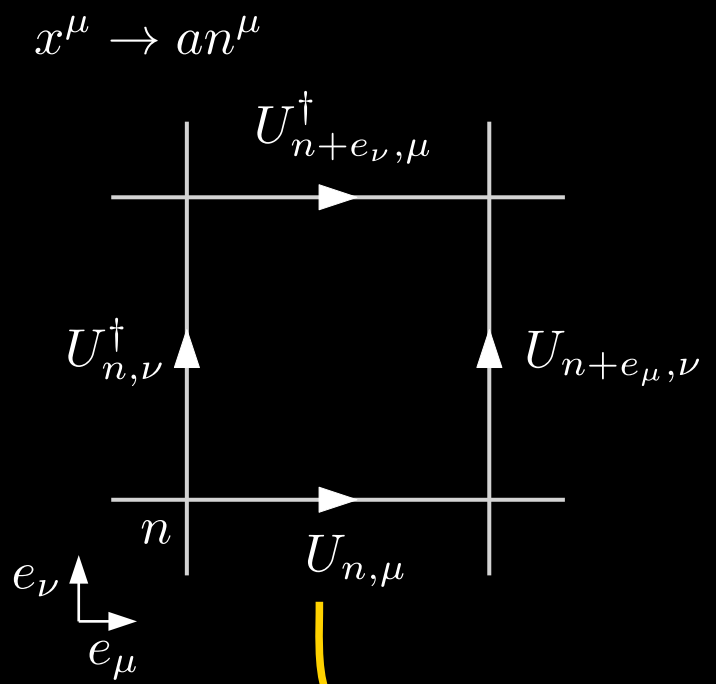
How to make these predictions?

- Nonperturbative problems
  - → **Numerically simulate** QCD degrees of freedom



Conjectured phase diagram credit: G. Endrődi J.Phys.Conf.Ser. 503 (2014) 012009

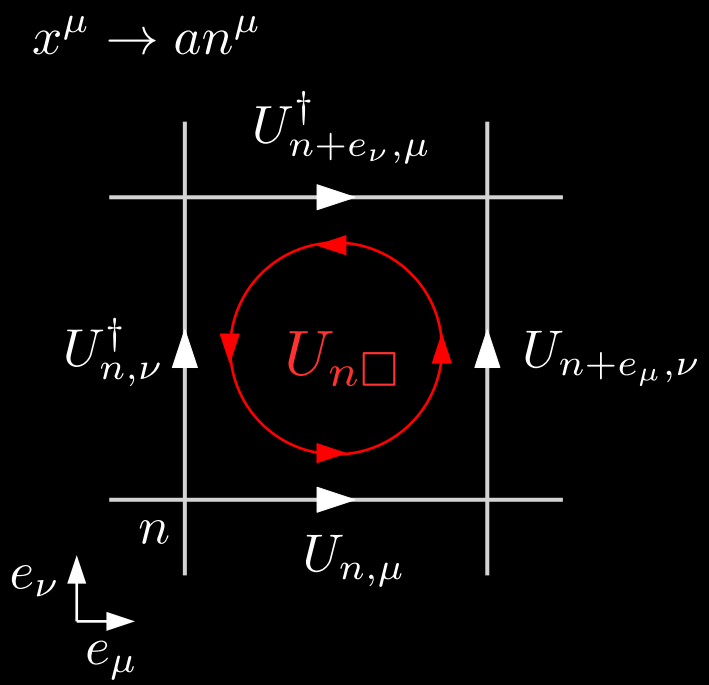
Traditional lattice field theory



$$\begin{pmatrix} -0.7485 & -0.2744 - 0.6037i \\ 0.2744 - 0.6037i & -0.7485 \end{pmatrix}$$

- Defines a field theory nonperturbatively
- Spacetime discretized with a lattice (e.g. square, cubic, hypercubic)
- Matter particles such as quarks are described “live” on the sites
- Gauge bosons live on oriented links joining sites
- Gauge fields belonging to some Lie group–the “gauge group” *G*

Traditional lattice field theory



Wilson's gauge action, S_W

"link operator" matrices in gauge group G

$$S_W = -\beta \sum_{n,\mu} \mathrm{tr}(\underbrace{U_{n,\mu} U_{n+e_\mu,\nu} U^\dagger_{n+e_\nu,\mu} U^\dagger_{n,\nu}}_{U_\square} + U^\dagger_\square)$$

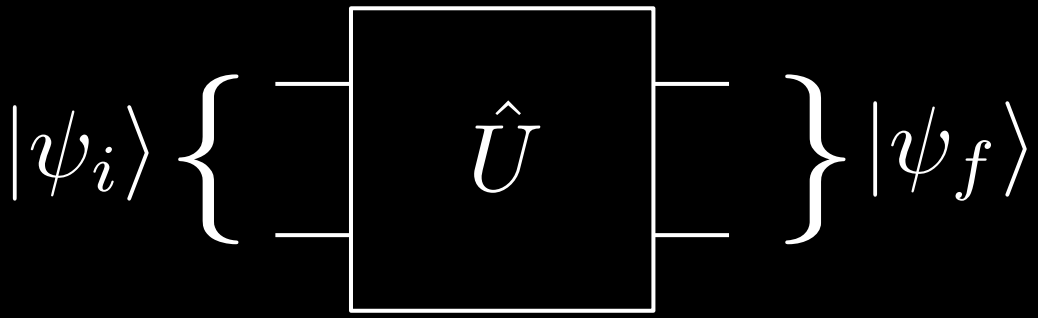
"plaquette" operator

for non-Abelian

- In classical simulations, exp(-S_W) acts like a probably weight for the configuration
- Real-time dynamics and nonzero baryon density both suffer from 'sign problems' in classical simulations

Classical problems.. quantum solutions?

Digital quantum computers:



- Unitary gates: $e^{-it\hat{H}}$ with Hamiltonian of interest
- Want to simulate gauge theory nonperturbatively
  - Gauge theory on the lattice
  - Hamiltonian lattice gauge theory
- Has no apparent sign problems

General problem:

How to map a Hilbert space $\mathcal{H}$, and $\hat{H}$, on to qubits & quantum gates?

Outline

- Digital quantum simulation basics
- Hamiltonian lattice gauge theory basics
- Model: SU(2), 1+1, staggered fermions
  - Mass, electric, and hopping propagators
  - Costs
  - Loop-string-hadron reformulation and comparison
- Conclusion

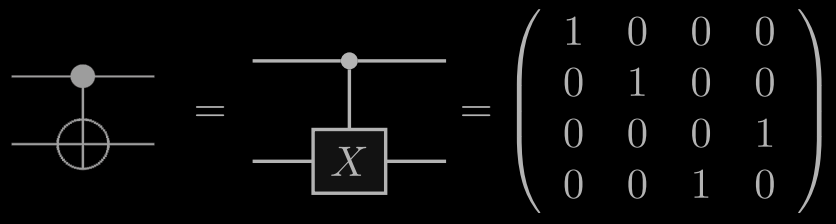
Digital quantum simulation: Primitives

Qubits: Two-state quantum systems

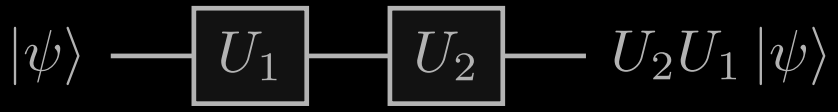
$$|\uparrow\rangle \leftrightarrow |0\rangle$$
$$|\downarrow\rangle \leftrightarrow |1\rangle$$

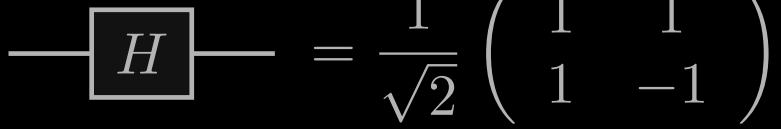
"computational basis"

Two-qubit computational basis:
|00>, |01>, |10>, |11>

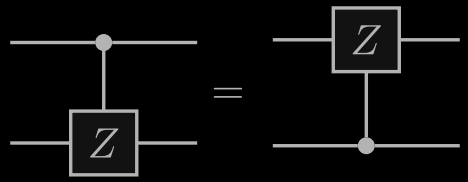


$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$





$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Nielsen & Chuang (2001)

Digital quantum simulation: Frameworks

‘Near-term’ and ‘far-term’ algorithms

NISQ era, near term

- Fewer qubits
- Limited connectivity
- No error correction
- Entangling gates (CNOT) costly

Credit: Google, Erik Lucero

Fault tolerant regime, far term

- Plenty of qubits
- High connectivity
- Error correction
- Non-Clifford operations (T gate) costly

Digital quantum simulation: Work flow

Three main steps

1. Initial state preparation
2. Time evolution
3. Measurements

This work: Time evolution only

- time evolution can be part of state preparation or measurements

Digital quantum simulation: Time evolution

Trotterization: Evolve for time *t* in *s* steps,

$$e^{-i\,t\,H} = \left(e^{-i\,\frac{t}{s}\,H}\right)^{s}$$

Product formulas: Approximate exponential of a sum by product of exponentials

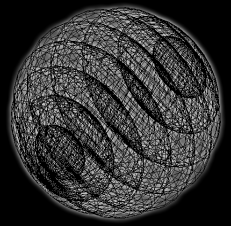
$$e^{-i\,\delta t\,\sum_k H_k} \simeq \prod_k e^{-i\,\delta t\,H_k}$$

Simplest case: Same ordering of H_k in every step
Generalizations: Higher-order Trotter; randomized

Hamiltonian lattice gauge theory

- Temporal gauge, continuous-time limit → Kogut-Susskind Hamiltonian formulation
- Gauge fields on spatial links with on-link Hilbert spaces
- E.g., SU(2)

Phys. Rev. D 11, 395 (1975)

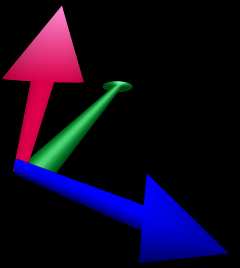


$$\langle g | j, m, m' \rangle = \sqrt{\frac{d_j}{|G|}} D^{(j)}_{m,m'}(g)$$

group-element basis

irrep basis

Left and right electric fields each have color-charge components, in addition to spatial components



Gauge transformations: $\hat{U}_{n,i} \to \Omega_n \hat{U}_{n,i} \Omega^\dagger_{n+e_i}$

- Rotations from the left (Ω_n) and right (Ω_{n+ei}) are generated by “left” and “right” electric fields

$$[\hat{E}^\alpha_{L/R}, \hat{E}^\beta_{L/R}] = i f^{\alpha\beta\gamma} \hat{E}^\gamma_{L/R}$$
$$[\hat{E}^\alpha_R, \hat{U}_{mm'}] = \left(\hat{U} T^\alpha\right)_{mm'}$$
$$[\hat{E}^\alpha_L, \hat{U}_{mm'}] = -\left(T^\alpha \hat{U}\right)_{mm'}$$

canonical commutation relations for a link

Hamiltonian lattice gauge theory

A feel for the on-link operators and states of SU(2):

$$E_R^3 \left|j, M, M'\right\rangle = M' \left|j, M, M'\right\rangle$$
$$E_R^\alpha E_R^\alpha \left|j, M, M'\right\rangle = j(j+1) \left|j, M, M'\right\rangle$$

$$\hat{H}_E = \frac{g^2}{2} \sum_{n,i} \hat{E}^\alpha_{n,i} \hat{E}^\alpha_{n,i}$$

$$U_{m,m'} \left|j, M, M'\right\rangle =$$
$$C_+(j, m, m', M, M') \left|j + 1/2, M + m, M' + m'\right\rangle$$
$$+ C_-(j, m, m', M, M') \left|j - 1/2, M + m, M' + m'\right\rangle$$

Off-diagonal SU(2) link operator

$$\hat{H}_B = -\sum_n \frac{1}{2g^2} \mathrm{tr}(\hat{U}_{n,\square} + \hat{U}^\dagger_{n,\square})$$

Hamiltonian lattice gauge theory

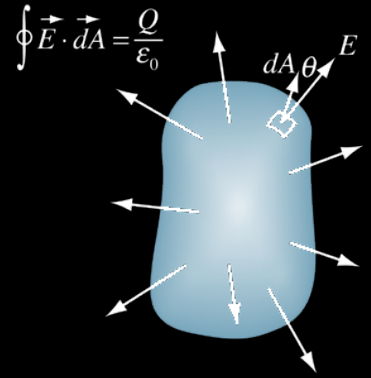
Plus Gauss law constraints

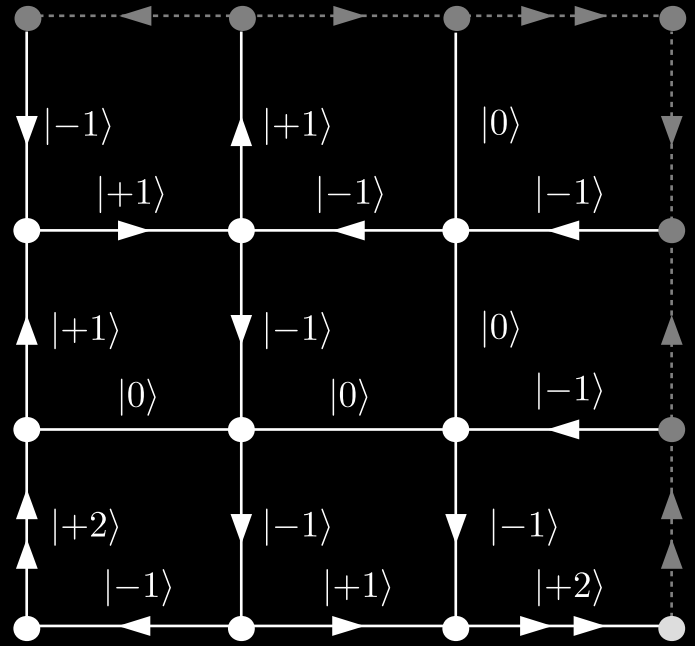
U(1) $\nabla \cdot \mathbf{E} - \rho = 0$

$\hat{\mathcal{G}}_n$ $\rho = \psi^\dagger \psi$

SU(N) $\mathbf{D} \cdot \mathbf{E}^a - \rho^a = 0$

$\hat{\mathcal{G}}_n^a$ $\rho^a = \psi^\dagger T^a \psi$





compact U(1)
electric eigenbasis

Gauss's law ⟷ gauge invariance

charge conservation

Formulation, basis considerations

- Different bases/formulations → different costs
- For gauge theories in particular, numerous *formulations*
- Examples:
  - Kogut-Susskind, electric or group-element basis *Zohar, NuQS collab., et al.*
  - dual or magnetic variables *Kaplan, JRS, Bauer, Grabowska*
    & tensor formulations *Meurice, Unmuth-Yockey, et al.*
  - purely fermionic formulation *Atas, J. Zhang, IQuS@UW group, Powell, et al.*
  - local-multiplet basis *Klco, Savage, JRS, Ciavarella*
  - gauge magnets/quantum link models *Wiese, Chandrasekharan, et al.*
  - prepotential/Schwinger boson formulations *Mathur, Anishetty, Raychowdhury, et al.*
  - loop-string-hadron formulation *Raychowdhury, JRS, Dasgupta, Kadam*
  - light-front formulation *Kreshchuk, Kirby, Love, Yao, et al.*
  - qubit models *Chandrasekharan, Singh, et al.*
- Most common basis choice: **Electric/irrep**

Formulation, basis considerations

Electric-basis pros

- States naturally discretized (for compact Lie groups)
- Gauss's law a function of electric fields
- Natural "UV" truncation scheme
  - Easily translates to truncating operators

Electric-basis cons

- Better-suited to strong coupling (opposite of continuum QCD)
- Many off-diagonal operators in 3+1 Hamiltonian

Formulation, basis considerations

Group-element basis pros

- Link operators are diagonalized
- No Clebsch-Gordon coefficients
- Well-suited for weak-coupling limit

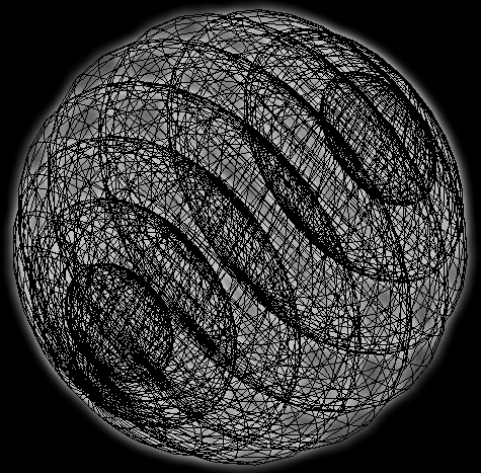
A detail of Spinoza monument in Amsterdam. © Dmitry Feichtner-Kozlov

Group-element basis cons

- Limited number of regular subgroups for SU(N)
  - Limited “resolution” with subgroups
  - 120 elements for SU(2)
  - 1080 for SU(3)
- Subsets generally do not preserve gauge symmetry
- Electric fields become tricky (next slide)

Model: SU(2), 1+1, staggered fermions

- Prototype non-Abelian gauge theory: SU(2)
  - Nontrivial representation theory
  - Similar complications to SU(3), fewer DOFs
- Keep the 1+1-D gauge fields
  - Remain representative of D>1+1
- Matter: fundamental ‘quarks’
- Involves link operators but lacks plaquettes



Model: SU(2), 1+1, staggered fermions

$$H = \sum_r E^\alpha(r) E^\alpha(r) \quad \} H_E$$

E^α : chromoelectric field

$\alpha = 1, 2, 3$ (adjoint rep)

bare mass

$$+ \mu \sum_r (-)^r \psi^\dagger(r)\psi(r) \quad \} H_M$$

$\psi(r) = \begin{pmatrix} \psi_1(r) \\ \psi_2(r) \end{pmatrix}$ fermionic color components

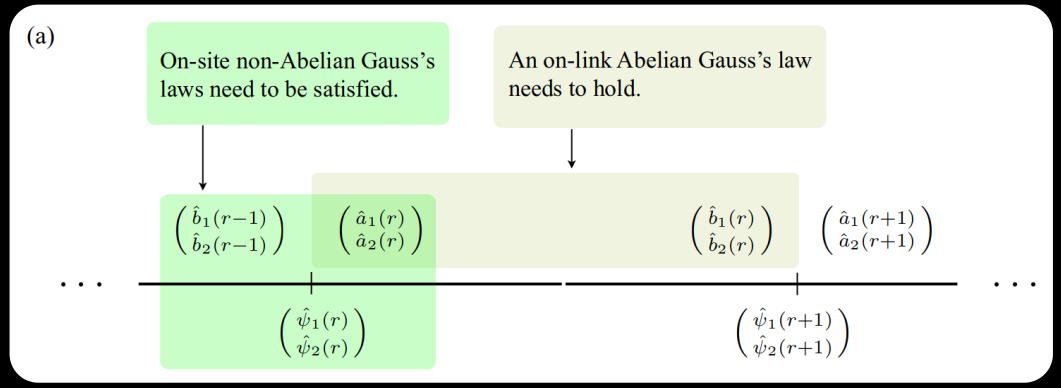
interaction strength

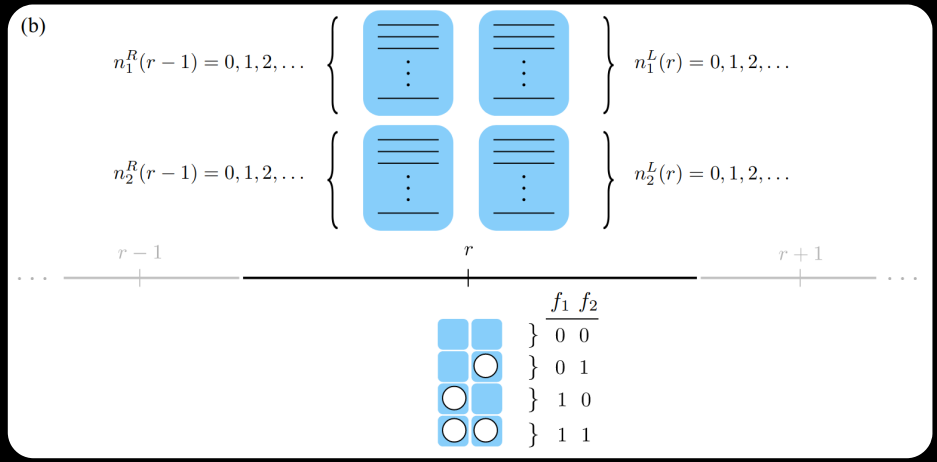
$$+ x \sum_r \psi^\dagger(r)\psi(r+1)U(r) + \text{H.c.} \quad \} H_I$$

$\psi^\dagger(r)U(r)\psi(r+1) + \text{H.c.}$: minimally-coupled “hopping term”

- We consider *Schwinger boson (SB)* formulation
  - SB ~ Kogut-Susskind, but more symmetric Hilbert space
  - Kogut-Susskind considered by Kan & Nam (arXiv:2107.12769)

Model: SU(2), 1+1, staggered fermions





Schwinger boson DOFs

Trotterization & diagonal terms H_M, H_E

$$\exp(-i\,\delta t(H_M + H_E + H_I)) \simeq \prod_r e^{-i\,\delta t H_I(r)} e^{-i\,\delta t H_E(r)} e^{-i\,\delta t H_M(r)}$$

$H_M,\ H_E$: Diagonal, “easy” to decompose with elementary gates exactly **(easy != efficient)**

$$E^\alpha E^\alpha \left|j, m_L, m_R\right\rangle_{KS} = j(j+1) \left|j, m_L, m_R\right\rangle_{KS}$$

Basic idea: Compute the function of occupation numbers into auxiliary register, then use “phase kickback” to effect $e^{-i\,\delta t E^\alpha E^\alpha}$

Electric propagators

Diagonal “easy,” given a function *f(n)* and phase kickback:

$$\mathcal{U}_f \left|n\right\rangle \left|0\right\rangle_{\text{aux}} = \left|n\right\rangle \left|f(n)\right\rangle_{\text{aux}}$$

$$e^{i\phi\hat{N}_{\text{aux}}} \left|n\right\rangle \left|f(n)\right\rangle_{\text{aux}} = e^{i\phi f(n)} \left|n\right\rangle \left|f(n)\right\rangle_{\text{aux}}$$

$$\mathcal{U}_f^\dagger e^{i\phi f(n)} \left|n\right\rangle \left|f(n)\right\rangle_{\text{aux}} = e^{i\phi f(n)} \left|n\right\rangle \left|0\right\rangle_{\text{aux}}$$

Can still be inefficient – evaluating *f(n)* may be costly

$$E^\alpha E^\alpha \left|j, m_L, m_R\right\rangle_{KS} = j(j+1) \left|j, m_L, m_R\right\rangle_{KS}$$

Lattice gauge theory H_E only calls for addition and multiplication
Cost of *j(j+1)* dominated by a single multiplier

Hopping terms

$$H_{\text{hop}}/x = \sum_{m,n} \psi_m^\dagger \chi_n U_{mn} + \text{H.c.}$$

(Schwinger boson formulation)

$$U = \frac{1}{\sqrt{a^\dagger \cdot a + 1}} \begin{pmatrix} -a_1 b_2 + a_2^\dagger b_1^\dagger & a_1 b_1 + a_2^\dagger b_2^\dagger \\ -a_2 b_2 - a_1^\dagger b_1^\dagger & a_2 b_1 - a_1^\dagger b_2^\dagger \end{pmatrix} \frac{1}{\sqrt{a^\dagger \cdot a + 1}}$$

$$\equiv \begin{pmatrix} -A_1 b_2 + A_2^\dagger b_1^\dagger & A_1 b_1 + A_2^\dagger b_2^\dagger \\ -A_2 b_2 - A_1^\dagger b_1^\dagger & A_2 b_1 - A_1^\dagger b_2^\dagger \end{pmatrix}$$

(absorbed inverse roots into A_k)

$$H_{\text{hop}} = \Sigma_{j=1}^{8} H_{\text{hop}}^{(j)} \qquad \nu = 8$$

$$H_{\text{hop}}^{(8)}/x = -\sigma_2^+ A_1 \sigma_3^- b_1 + \text{H.c.}$$

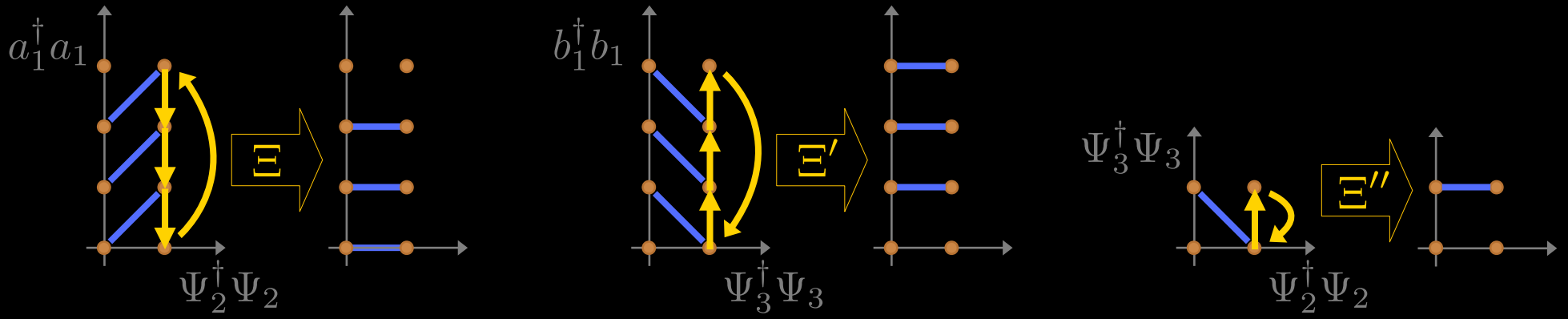
$$[H_I^{(i)}(r), H_I^{(j)}(r)] \neq 0$$

- G^3 conserved by each subterm (G^3 eigenbasis)
- Abelian Gauss law conserved too

Hopping (sub)terms

Three *shears** to prepare for rotation:

$$H^{(8)}_{\text{hop}}/x = -\sigma_2^+ A_1 \sigma_3^- b_1 + \text{H.c.}$$



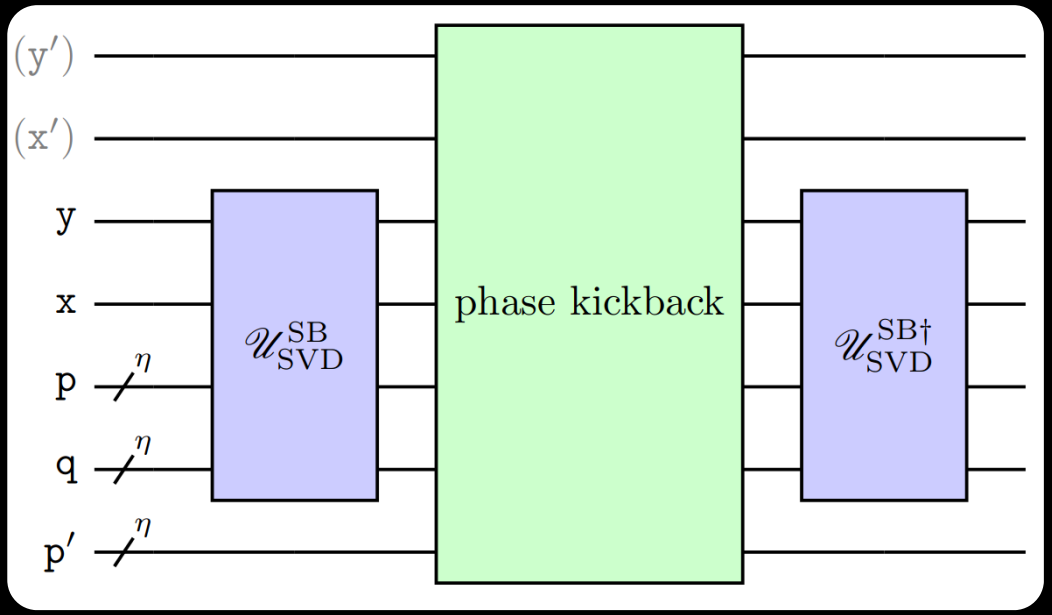
$$-\sigma_2^+ A_1 \sigma_3^- b_1 + \text{H.c.} \xrightarrow{H_2 \Xi'' \Xi' \Xi} -Z_2 \left|1\right\rangle\left\langle 1\right|_3 \sqrt{\frac{(a_1^\dagger a_1) b_1^\dagger b_1}{(a^\dagger \cdot a)(a^\dagger \cdot a + 1)}}$$

- *Z*-rotation on fermionic qubit 2, controlled by fermionic qubit 3, with phase depending on three occupation numbers of a_1, a_2, b_1
- The reduction from 64 → 8 simulated terms improves the Trotterization error bound

* JRS. [2105.11548]

Hopping terms

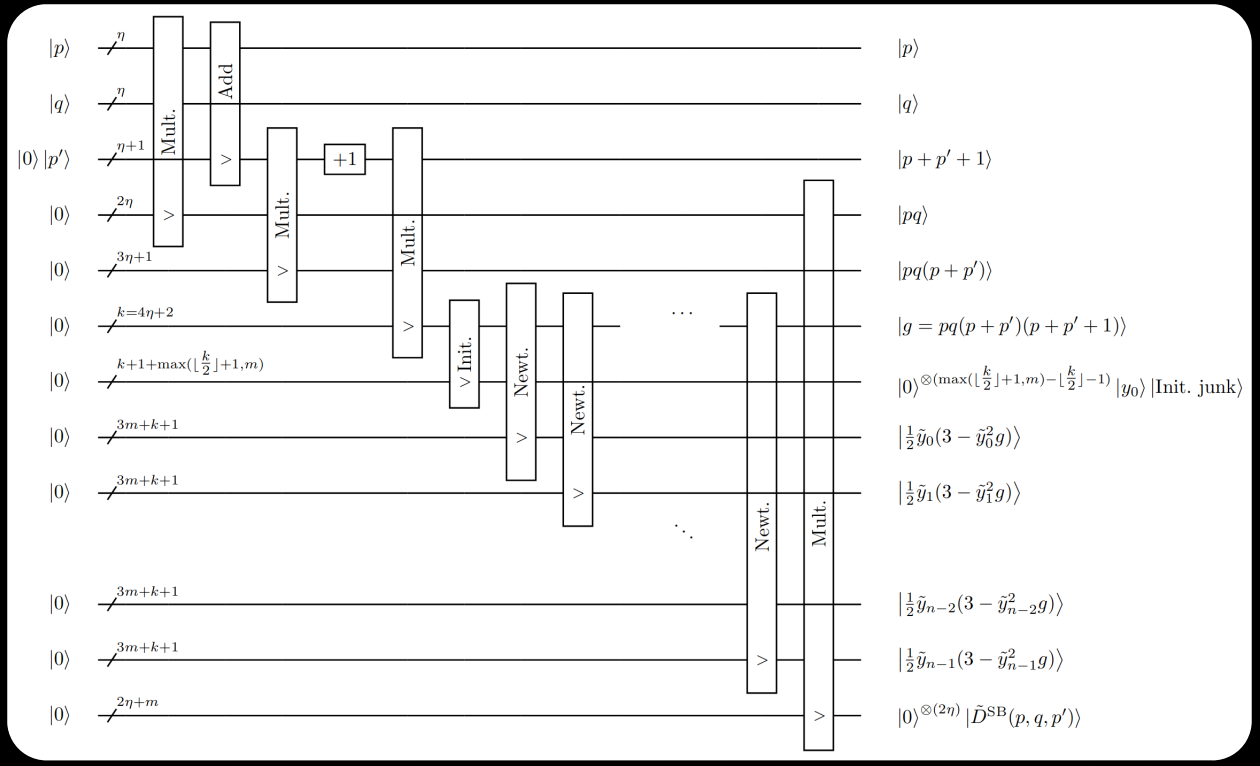
Simulating any one hopping subterm



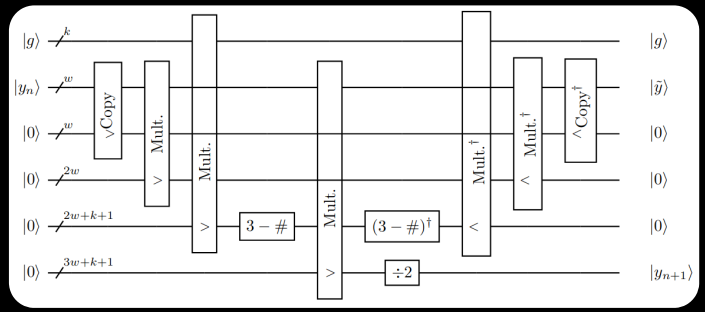
Instead of j(j+1) or similar, this requires evaluating

$$\mathcal{D}^{\mathrm{SB}}(p,q,p') \equiv \sqrt{\frac{p\,q}{(p+p')(p+p'+1)}}$$

Hopping terms



Phase evaluation circuit involving multiple rounds of Newton's method iterations



Single iteration of Newton's method involving numerous multipliers of increasing size

Resource costs of SB formulation

- Cost of simulation scales with desired error
- One metric: Spectral norm error of time-evolution operator
- Sources of error
  - Trotterization
  - Truncated function evaluation
  - Imperfect rotation gates (“synthesis” error)

Resource costs of SB formulation

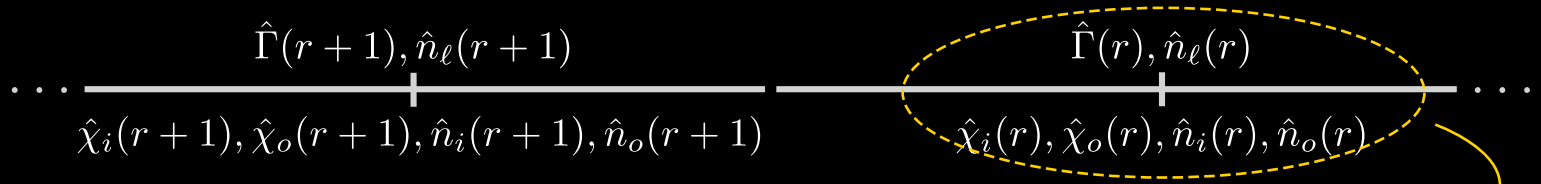
m/g	x	η	L	t/a_s	Δ	$\alpha_{\text{Trot.}}$	$\alpha_{\text{Newt.}}$	Qubits	T gates
1	1	4	100	1	0.01	90%	9%	2626	8.19713×10^{11}
1	1	4	100	1	0.001	90%	9%	2704	3.09951×10^{12}
1	1	4	100	10	0.01	90%	9%	2704	3.0993×10^{13}
1	1	4	100	10	0.001	90%	9%	2808	1.2146×10^{14}
1	1	4	1000	1	0.01	90%	9%	18904	3.12769×10^{13}
1	1	4	1000	1	0.001	90%	9%	19008	1.22564×10^{14}
1	1	4	1000	10	0.01	90%	9%	19008	1.22564×10^{15}
1	1	4	1000	10	0.001	90%	9%	19086	4.48657×10^{15}
1	1	8	100	1	0.01	90%	9%	4398	5.79224×10^{12}
1	1	8	100	1	0.001	90%	9%	4476	2.1482×10^{13}
1	1	8	100	10	0.01	90%	9%	4476	2.14816×10^{14}
1	1	8	100	10	0.001	90%	9%	4580	8.22615×10^{14}
1	1	8	1000	1	0.01	90%	9%	35076	2.16773×10^{14}
1	1	8	1000	1	0.001	90%	9%	35180	8.30098×10^{14}
1	1	8	1000	10	0.01	90%	9%	35180	8.30094×10^{15}
1	1	8	1000	10	0.001	90%	9%	35258	2.99214×10^{16}
1	10	4	100	1	0.01	90%	9%	2626	5.7715×10^{11}
1	10	4	100	1	0.001	90%	9%	2704	2.18285×10^{12}
1	10	4	100	10	0.01	90%	9%	2704	2.18258×10^{13}
1	10	4	100	10	0.001	90%	9%	2808	8.55326×10^{13}
1	10	4	1000	1	0.01	90%	9%	18904	2.2027×10^{13}
1	10	4	1000	1	0.001	90%	9%	19008	8.63137×10^{13}
1	10	4	1000	10	0.01	90%	9%	19008	8.63103×10^{14}
1	10	4	1000	10	0.001	90%	9%	19086	3.15948×10^{15}
1	10	8	100	1	0.01	90%	9%	4398	1.33288×10^{12}
1	10	8	100	1	0.001	90%	9%	4476	4.94102×10^{12}
1	10	8	100	10	0.01	90%	9%	4476	4.94053×10^{13}
1	10	8	100	10	0.001	90%	9%	4580	1.89192×10^{14}
1	10	8	1000	1	0.01	90%	9%	35076	4.98595×10^{13}
1	10	8	1000	1	0.001	90%	9%	35180	1.9092×10^{14}
1	10	8	1000	10	0.01	90%	9%	35180	1.90912×10^{15}
1	10	8	1000	10	0.001	90%	9%	35258	6.88164×10^{15}

Far-term simulation costs as a function of Hamiltonian parameters (m/g,x,L, and Λ=2^{η}-1), evolution time (t/a_s), and desired bound on the controlled sources of error (Δ). Qubit counts are the sum of qubits needed to represent lattice DOFs and ancilla qubits used for implementing the time evolution.

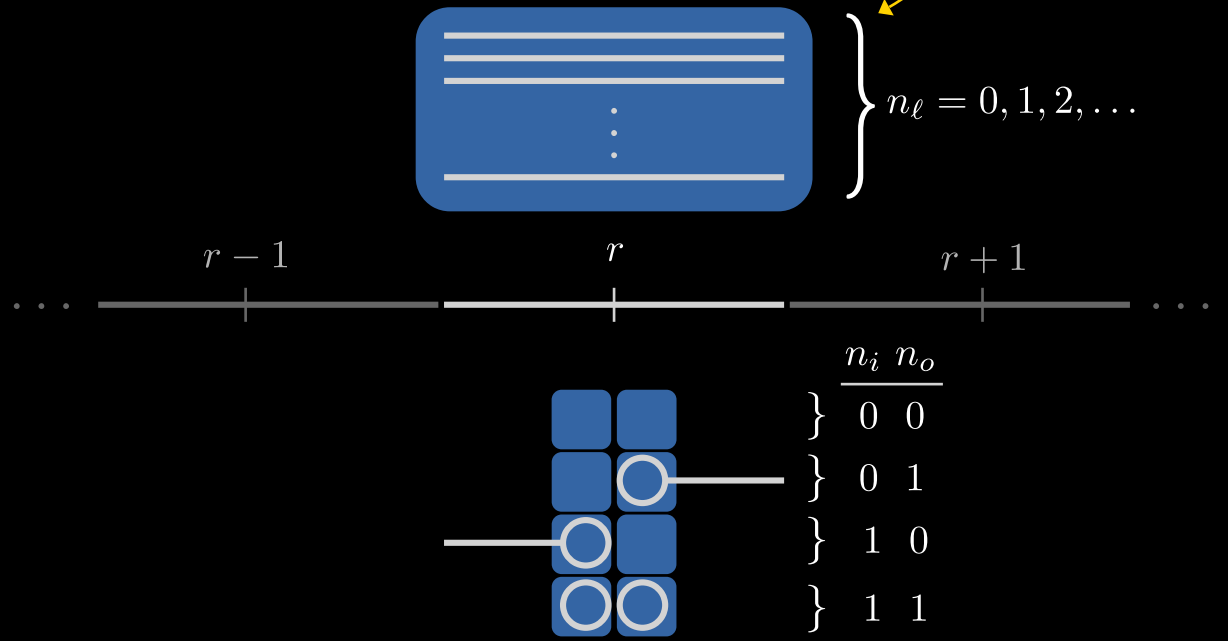
m/g	$\Delta_{\text{Trot.}}$	x	L	η	t/a_s	Qubits	Min. s	Min. CNOTs
1	10%	0.1	10	2	1	92	186	4.8613×10^{6}
1	10%	0.1	10	2	5	92	2072	5.41538×10^{7}
1	10%	0.1	10	4	1	164	433	5.21403×10^{8}
1	10%	0.1	10	4	5	164	4841	5.82936×10^{9}
1	10%	0.1	20	2	1	192	262	1.44561×10^{7}
1	10%	0.1	20	2	5	192	2929	1.61611×10^{8}
1	10%	0.1	20	4	1	344	613	1.55832×10^{9}
1	10%	0.1	20	4	5	344	6846	1.74034×10^{10}
1	10%	1	10	2	1	92	102	2.66587×10^{6}
1	10%	1	10	2	5	92	1133	2.96121×10^{7}
1	10%	1	10	4	1	164	129	1.55337×10^{8}
1	10%	1	10	4	5	164	1432	1.72436×10^{9}
1	10%	1	20	2	1	192	144	7.94534×10^{6}
1	10%	1	20	2	5	192	1602	8.8392×10^{7}
1	10%	1	20	4	1	344	182	4.62667×10^{8}
1	10%	1	20	4	5	344	2024	5.14526×10^{9}
1	5%	0.1	10	2	1	92	262	6.84763×10^{6}
1	5%	0.1	10	2	5	92	2929	7.65523×10^{7}
1	5%	0.1	10	4	1	164	613	7.38153×10^{8}
1	5%	0.1	10	4	5	164	6846	8.24371×10^{9}
1	5%	0.1	20	2	1	192	371	2.04703×10^{7}
1	5%	0.1	20	2	5	192	4143	2.28594×10^{8}
1	5%	0.1	20	4	1	344	866	2.20148×10^{9}
1	5%	0.1	20	4	5	344	9682	2.46128×10^{10}
1	5%	1	10	2	1	92	144	3.76358×10^{6}
1	5%	1	10	2	5	92	1602	4.18699×10^{7}
1	5%	1	10	4	1	164	182	2.19158×10^{8}
1	5%	1	10	4	5	164	2024	2.43723×10^{9}
1	5%	1	20	2	1	192	203	1.12007×10^{7}
1	5%	1	20	2	5	192	2266	1.25029×10^{8}
1	5%	1	20	4	1	344	257	6.53326×10^{8}
1	5%	1	20	4	5	344	2863	7.2781×10^{9}

Near-term simulation costs as a function of Hamiltonian parameters (m/g,x,L, and η), evolution time (t/a_s=2 x T), and desired bound on the controlled sources of error (Δ_{Trot}). Qubit counts are the register size of the lattice and exclude possible ancilla qubits (which are insignificant in the near-term circuits cost). Other tabulated costs are the minimal required number of second-order Trotter steps (based on our second-order Trotterization scheme) and the associated CNOT-gate count (for the naive circuitization approach based on the full Pauli decomposition of diagonal phase functions) in the zero-noise limit.

LSH reformulation



Loop-string-hadron formulation is derived from SB formulation but uses fewer bosonic DOFs per site. Every field is strictly SU(2) invariant



LSH reformulation

LSH operators define an SU(2)-singlet basis

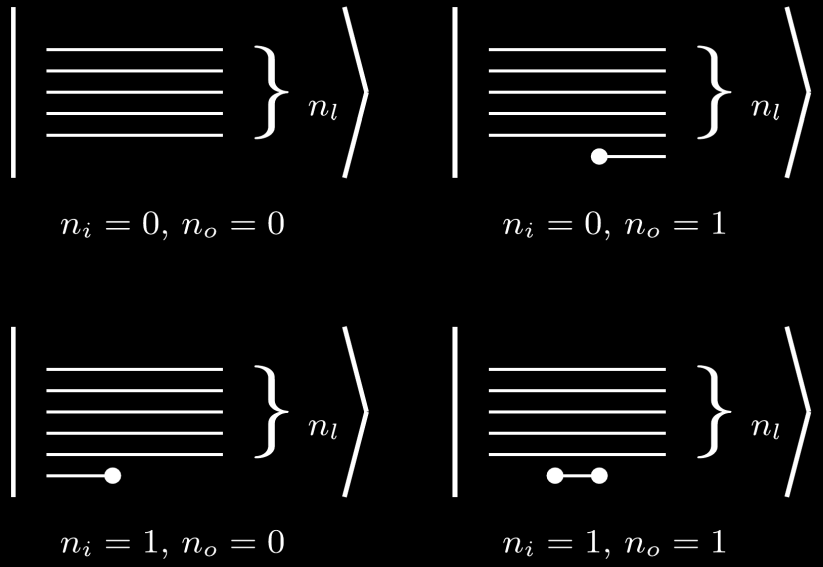
- Take a reference state, e.g., 0 flux & 0 fermions
- Act locally with any product of LSH operators
- Result is SU(2)-invariant

$$||n_l, n_i = 0, n_o = 0\rangle \equiv (\mathcal{L}^{++})^{n_l}|0\rangle$$

$$||n_l, n_i = 0, n_o = 1\rangle \equiv (\mathcal{L}^{++})^{n_l}\mathcal{S}^{++}_{\text{out}}|0\rangle$$

$$||n_l, n_i = 1, n_o = 0\rangle \equiv (\mathcal{L}^{++})^{n_l}\mathcal{S}^{++}_{\text{in}}|0\rangle$$

$$||n_l, n_i = 1, n_o = 1\rangle \equiv (\mathcal{L}^{++})^{n_l}\mathcal{H}^{++}|0\rangle$$



$$\mathcal{N}_\psi = \mathcal{N}_i + \mathcal{N}_o$$

$$\mathcal{N}_L = \mathcal{N}_l + \mathcal{N}_o(1 - \mathcal{N}_i)$$

$$\mathcal{N}_R = \mathcal{N}_l + \mathcal{N}_i(1 - \mathcal{N}_o)$$

LSH reformulation

Easy terms

$$\hat{H}_M \to m_0 \sum_x (-)^x \mathcal{N}_\psi(x)$$

$$\hat{H}_E \to \frac{g_0^2}{4} \sum_x \left[\frac{1}{2}\mathcal{N}_R(x)\left(\frac{1}{2}\mathcal{N}_R(x)+1\right) + \frac{1}{2}\mathcal{N}_L(x)\left(\frac{1}{2}\mathcal{N}_L(x)+1\right)\right]$$

Hard terms

$$\hat{H}_I \to \sum_x \frac{1}{\sqrt{\mathcal{N}_L(x)+1}} \left[\sum_{\sigma=\pm} \mathcal{S}_{\text{out}}^{+,\sigma}(x)\mathcal{S}_{\text{in}}^{\sigma,-}(x+1)\right] \times \frac{1}{\sqrt{\mathcal{N}_R(x+1)+1}} + \text{H.c.}$$

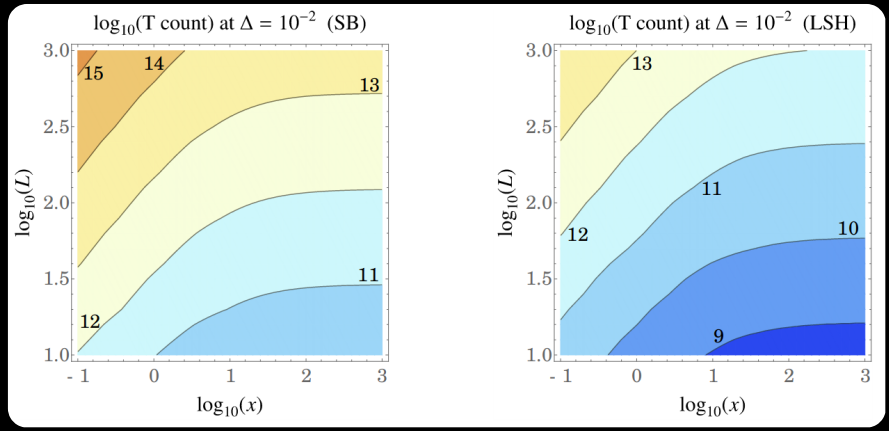
$$\hat{\psi}^\dagger(x)\hat{U}_L(x) = \frac{1}{\sqrt{\mathcal{N}_L(x)+1}} \begin{pmatrix} \mathcal{S}_{\text{out}}^{++}(x), & \mathcal{S}_{\text{out}}^{+-}(x) \end{pmatrix},$$

$$\hat{U}_R(x)\hat{\psi}(x) = \begin{pmatrix} \mathcal{S}_{\text{in}}^{+-}(x) \\ \mathcal{S}_{\text{in}}^{--}(x) \end{pmatrix} \frac{1}{\sqrt{\mathcal{N}_R(x)+1}}.$$

Important: $\nu = 2$

SU(2) LSH vs Schwinger bosons

							Schwinger bosons		LSH	
x	η	L	t/a_s	Δ	$\alpha_{\text{Trot.}}$	$\alpha_{\text{Newt.}}$	Qubits	T gates	Qubits	T gates
1	4	100	1	0.01	90%	9%	2626	8.19713×10^{11}	1319	3.91817×10^{10}
1	4	100	1	0.001	90%	9%	2704	3.09951×10^{12}	1397	1.5172×10^{11}
1	4	100	10	0.01	90%	9%	2704	3.0993×10^{13}	1397	1.51643×10^{12}
1	4	100	10	0.001	90%	9%	2808	1.2146×10^{14}	1475	5.76229×10^{12}
1	4	1000	1	0.01	90%	9%	18904	3.12769×10^{13}	6797	1.53099×10^{12}
1	4	1000	1	0.001	90%	9%	19008	1.22564×10^{14}	6875	5.81562×10^{12}
1	4	1000	10	0.01	90%	9%	19008	1.22564×10^{15}	6875	5.81468×10^{13}
1	4	1000	10	0.001	90%	9%	19086	4.48657×10^{15}	6979	2.29217×10^{14}
1	8	100	1	0.01	90%	9%	4398	5.79224×10^{12}	1807	2.72735×10^{11}
1	8	100	1	0.001	90%	9%	4476	2.1482×10^{13}	1885	1.03709×10^{12}
1	8	100	10	0.01	90%	9%	4476	2.14816×10^{14}	1885	1.03705×10^{13}
1	8	100	10	0.001	90%	9%	4580	8.22615×10^{14}	1963	3.87886×10^{13}
1	8	1000	1	0.01	90%	9%	35076	2.16773×10^{14}	10885	1.04652×10^{13}
1	8	1000	1	0.001	90%	9%	35180	8.30098×10^{14}	10963	3.91414×10^{13}
1	8	1000	10	0.01	90%	9%	35180	8.30094×10^{15}	10963	3.91412×10^{14}
1	8	1000	10	0.001	90%	9%	35258	2.99214×10^{16}	11067	1.5154×10^{15}



T-gate costs at fixed m/g=1. Other simulation parameters not explicitly shown are η = 8, t/a_s = 1, $\alpha_{\text{Trot.}}$ = 90%, $\alpha_{\text{Newt.}}$ = 9%, and $\alpha_{\text{synth.}}$ = 1%.

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arXiv:2212.14030

~20x T gate reduction with LSH

Conclusions

- Reproducing Clebsch-Gordon coefficients for non-Abelian gauge links dominates the circuit cost
- LSH formulation can give significant pre-factor savings (or even better scaling) over Schwinger-boson/Kogut-Susskind formulation
- Splitting of costly terms impacts both number of costly subroutines and size of calculated error bound
- Seeking circuitizable subterms that conserve symmetries can lead to more efficient splittings

FIN

Thank you for your attention